Assam Academy of Mathematics
MATHEMATICS OLYMPIAD - 2014

CATEGORY – IV
(Classes XI and XII)

Marks : $10 \times 10 = 100$ **Time : 11 am to 2 pm**

Each problem carries 10 marks.

1. Prove that, if a pentagon (five-sided polygon) inscribed in a circle has equal angles, then its sides are equal.

2. Prove that the product of four consecutive natural numbers can not be the square of an integer.

3. Find the sum of all distinct four-digit numbers that contain only the digit 1, 2, 3, 4, 5 each atmost once.

4. Determine all positive integers n for which $2^n + 1$ is divisible by 3.

5. Let n be a positive integer. Prove that the number $a_n = 2^{2^n} - 1$ has atleast n distinct prime divisors.

6. Prove that two of the four roots of the polynomial
$x^4 + 12x - 5$ add upto 2.

7. Prove that in an arbitrary triangle, the sum of the lengths of the altitudes is less than the triangle's perimeter.

8. Suppose that in the quadrilateral ABCD we inscribe a circle with centre O. Prove that the sum of angles $\angle AOB$ and $\angle COD$ equals $180°$.

9. Let $a, b, c \in \mathbb{R}$, $a \neq 0$, such that a and $4a + 3b + 2c$ have the same sign. Show that the equation $ax^2 + bx + c = 0$ can not have both roots in the interval $(1, 2)$.

[Hints : $0 \leq \frac{1}{a}(4a + 3b + 2c)$. Then $0 \leq (\alpha - 1)(\beta - 2) + (\alpha + 2)(\beta - 1)$. Leads

(turn over)

to a contradiction.]

10. Show that there does not exist a function $f:\mathrm{N}\to\mathrm{N}$ which satisfy

(a) $f(2)=3$;

(b) $f(mn)=f(m)f(n)$ for all m,n in N.

(c) $f(m)<f(n)$ where $m<n$.

[Hints Suppose the contrary. Arrive at $256<243$]