

PROBLEMS IN ELEMENTARY NUMBER THEORY

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1. Let a be a positive integer and p be a prime. Then prove that $a^p \equiv a \pmod{p}$.
Fermat
2. Is $4^{545} + 545^4$ a prime?
Russia
3. If $n > 1$, then prove that $n^4 + 4^n$ is never a prime.
Kürschak, 1978
4. Can a number, A consisting of 600 sixes and some zeroes be a square?
5. Prove that the equation $15x^2 - 7y^2 = 9$ has no integral solutions.
6. Show that the equation $x^2 + y^2 + z^2 = 2xyz$ has no integral solutions except $x = y = z = 0$.
7. Prove that a number with 3^n equal digits is divisible by 3^n .
8. How many zeroes are at the end of $2012!$?
9. Prove that $1000 \dots 001$ with 1961 zeroes is composite.
10. If $n \in \mathbb{N}$ and $3n + 1$ and $4n + 1$ are perfect squares, then prove that $56 \mid n$.

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