

General Mathematics
Test 2

Time: 4 Hours

Answer as many as you can

Maximum Marks: 120

Questions numbered 1, 3, 4, 6, 10, 17, 18, 19, 21, 37 and 38 are compulsory.

Pass marks: 40

1. Give examples of pairs of linear equations in two variables which intersect each other at a point and which do not intersect each other at any point. Can there be a pair of linear equations in two variables which intersect each other at two points? Explain briefly. **(1+1+1 marks)**
2. Solve the following pair of equations geometrically: $2x + 4y - 12 = 0, x + 2y - 4 = 0$. **(3 marks)**
3. Consider the two lines $ax + by + c = 0, dx + ey + f = 0$, write down the conditions for these two lines to be parallel to each other, to be coincident with each other and to be intersecting at some point. **(3 marks)**
4. What does it mean for a pair of lines to be consistent and inconsistent? Explain. **(2+1marks)**
5. Solve the following pairs of equations using the substitution method and the elimination method separately:
$$\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0.$$
(2+2 marks)
6. Solve for x and y :
$$\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11}.$$
(3 marks)
7. Solve for x and y : $\frac{x}{a} + \frac{y}{b} = a + b, \frac{x}{a^2} + \frac{y}{b^2} = 2$, where $a, b \neq 0$. **(3 marks)**
8. If $\frac{4x-3y}{7x-6y} = \frac{4}{13}$, find $\frac{x}{y}$. **(3 marks)**
9. Students are made to stand in rows. If one student is extra in each row there would be 2 rows less. If one student is less in each row there would be 3 rows more. Find the number of students in the class. **(4 marks)**
10. Solve the pair of equations $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$ using the cross multiplication method. Mention the conditions for these pairs of equations to have a unique solution, infinitely many solutions and no solutions. **(3+1+1+1 marks)**
11. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train. **(4 marks)**
12. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x . **(5 marks)**

13. Find out the roots of $ax^2 + bx + c = 0$, where $a, b, c \neq 0$ and integers. **(3 marks)**
14. Solve $9x^2 - 15x + 6 = 0$ by two different methods. **(3+3 marks)**
15. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. **(3 marks)**
16. The difference of squares of two numbers is 180. The square of the smaller number is eight times the larger number. Find the two numbers. **(3 marks)**
17. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. **(3 marks)**
18. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + px + q = 0$ share a common root, then find the quadratic equation which has as its roots the other roots (which are not common) of the previous two equations. **(4 marks)**
19. Solve for x : $x^{10} - x^9 - 2 = 0$. **(3 marks)**
20. The equation $x^2 + px + q = 0$ has integer solutions for some prime numbers p and q . Find the values of p and q when this happens. **(5 marks)**
21. Solve $4^x - 4.2^x + 4 = 0$ for values of x . **(3 marks)**
22. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. Also state and prove the converse. **(4+3 marks)**
23. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. **(2 marks)**
24. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. **(2 marks)**
25. If in two triangles, corresponding angles are equal, then prove that their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar. **(4 marks)**
26. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then prove that the two triangles are similar. **(3 marks)**
27. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds. **(3 marks)**
28. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of triangle PQR . Show that the two triangles are similar to each other. **(4 marks)**
29. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. **(4 marks)**
30. If the areas of two similar triangles are equal, prove that they are congruent. **(3 marks)**

31. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals. **(3 marks)**
32. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians. **(3 marks)**
33. State and prove the Pythagoras' theorem. Also state and prove it's converse. **(3+2 marks)**
34. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. **(3 marks)**
35. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. **(3 marks)**
36. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides. **(4 marks)**
37. In a triangle with side lengths a, b, c and lengths of corresponding medians m_a, m_b, m_c , prove that $m_a + m_b + m_c < a + b + c < \frac{4}{3}(m_a + m_b + m_c)$. **(4 marks)**
38. In triangle ABC , if D is the midpoint of BC then prove that

$$AB^2 + AC^2 = 2(AD^2 + CD^2).$$

(4 marks)

39. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides. **(4 marks)**
40. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals. **(3 marks)**
41. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals. **(3 marks)**