

AN INTRODUCTION TO INEQUALITIES

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1. PROPERTIES

- For any two real numbers a and b , we can have any one of the following three relations:
 - $a > b$
 - $a < b$
 - $a = b$
- No square is negative!

$$a^2 \geq 0, \forall a \in \mathbb{R}.$$

- A.M.-G.M. Inequality: For any two numbers a and b , we have $\frac{a+b}{2} \geq \sqrt{ab}$.

2. PROBLEMS

- Which of the fractions $\frac{4567890123}{780123456}$ and $\frac{4567890124}{780123458}$ is greater?
- A man receives $\frac{p}{q}$ th part of Rs. a and $\frac{q}{p}$ th part of Rs. a . He then gives away Rs. $2a$. Show that the man cannot lose in the transaction.
- Determine which of the two numbers 1000^{1000} and 1001^{999} is greater?
- Prove that $3^{200} > 2^{300}$.
- Which is larger, $9\sqrt{9!}$ or $10\sqrt{10!}$?
- If p and q are positive real numbers such that $p + q = 1$, then prove that

$$\left(p + \frac{1}{p}\right)^2 + \left(q + \frac{1}{q}\right)^2 \geq \frac{25}{2}.$$

- If a, b and c are three real, then show that $a^4 + b^4 + c^2 \geq 2\sqrt{2}abc$.
- Three positive numbers a, b and c satisfy $a \geq b \geq c$ and $a + b + c \geq 1$. Prove that $a^2 + 3b^2 + 5c^2 \leq 1$.
- If a, b and c are three real numbers such that $a^2 + b^2 + c^2 = 1$, then prove that

$$-\frac{1}{2} \leq ab + bc + ca \leq 1.$$

- Determine which of the following two numbers is bigger:

$$\sqrt{5 + \sqrt{21}} + \sqrt{8 + \sqrt{55}} \text{ and } \sqrt{7 + \sqrt{33}} + \sqrt{9 + \sqrt{65}}.$$

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