

POLYNOMIALS

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1. If $f(x)$ be a polynomial of degree n , then there exists polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)q(x) + r(x)$ for some polynomial $g(x)$ such that $\deg r < \deg g$ or $r(x) = 0$.

Division Algorithm

2. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ have integer co-efficients and let $z \in Z$, then $f(z) = 0$ if and only if $z \mid a_0$. Further if, $a_n = 1$, then each rational root of f is an integer.

Rational Roots Theorem

3. For any polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, the sum of the roots taken k at a time is $(-1)^k \frac{a_{n-k}}{a_n}$.

Viete's Theorem

4. Let x, y, z be the roots of $a^3 + 3a^2 - 7a + 1$. Then find the value of $x^2 + y^2 + z^2$.

5. For which $n \in N$ is $x^2 + x + 1 \mid x^{2n} + x^n + 1$?

6. For which n is $37 \mid 100 \dots (n \text{ times}) \dots 0001000 \dots (n \text{ times}) \dots 0001$?

7. Solve $x^5 + y^5 = 33, x + y = 3$.

8. For which $a \in R$ is the sum of the squares of the zeroes of $x^2 - (a - 2)x - a - 1$ minimal?

9. Show that $(x - 1)^2 \mid nx^{n+1} - (n + 1)x^n + 1$.

10. Show that $(x + 1)^2 \mid x^{4n+2} + x^{2n+1} + 1$.

11. Solve $x^4 + a^6 - 3ax^3 + 3a^3x = 0$.

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