Some Results on Generalized Multiplicative Perfect Numbers

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- n is called a **k-multiplicatively perfect number**, if $T(n) = n^k$ for $k \geq 2$.

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- If $n = p_1^{a_1} \cdots p_r^{a_r}$ is the prime factorization of $n > 1$, a divisor d of n is called an **exponential divisor** (or, *e*-divisor for short) if $d = p_1^{b_1} \cdots p_r^{b_r}$ with $b_i \mid a_i$ for $i = 1, \ldots, r$.

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- Let $T_e(n)$ denote the product of all the e-divisors of n. Then n is called **multiplicatively e-perfect** if $T_e(n) = n^2$ and **multiplicatively e-superperfect** if $T_e(T_e(n)) = n^2$.

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Theorem (Sándor)

n is multiplicatively e-perfect if and only if $n = p^a$, where p is a prime and a is an ordinary perfect number. n is multiplicatively e-superperfect if and only if $n = p^a$, where p is a prime and a is an ordinary superperfect number.

k-multiplicatively e-perfect numbers

Theorem (Laugier – S)

If $n = p^a$, where p is a prime and a is a k-perfect number, then n is k-multiplicatively e-perfect. If $n = p^a$, where p is a prime and a is a k-superperfect number, then n is k-multiplicatively e-superperfect.

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- A number n is called a T_0^*T-**superperfect number** if $T^*(T(n)) = n^2$ and it is called a k-T_0^*T-**perfect number** if $T^*(T(n)) = n^k$ for $k \geq 2$.

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- They also introduced the **k-$T_0 T^*$-perfect numbers** as the numbers n such that $T(T^*(n)) = n^k$ for $k \geq 2$.

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- They also introduced the **k-T_0T^*-perfect numbers** as the numbers n such that $T(T^*(n)) = n^k$ for $k \geq 2$. It is our aim to characterize these k-T_0T^*-perfect numbers.

Characterization Results

Theorem (Laugier – S)

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9. *All* 10-$T_0 T^*$*-perfect numbers have the form* $n = p_1^{19}$.

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All p^2-T_0T^*-perfect numbers for a prime p have the form $n = p_1^{2p^2-1}$ or $n = p_1^{(p-1)/2}p_2^{(p-1)/2}$.

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Theorem (Laugier – S)

Let p be a prime, with $2^p - 1$ being a Mersenne prime. Then $2^{p-1}(2^p - 1)$ is the only even perfect number which is a $3(2p-1)$-T_0T^*-perfect number.

Questions

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Thank you for your attention.