

MATHEMATICAL OLYMPIAD 2011 CATEGORY III

ASSAM ACADEMY OF MATHEMATICS
04 SEPTEMBER 2011
ALL QUESTIONS CARRY EQUAL MARKS.

- In a certain year there were exactly four Fridays and exactly four Mondays in January. On what day of the week did the Republic Day fall that year.
 - Crossout 10 digits from the number
12345 12345 12345 12345 12345
so that the remaining number is as large as possible.
- Can we draw a closed path made up of 9 line segments, each of which intersects exactly one of the other segments?
 - The product of 22 integers is equal to 1. Show that their sum cannot be zero.
- There are five types of envelopes and four types of stamps in a Post Office. How many ways are there to buy an envelope and a stamp?
 - A triangle of area 1 has sides of lengths a , b and c where $a \geq b \geq c$. Prove that $b \geq \sqrt{2}$.
- Prove that $\frac{a^2}{4} + b^2 + c^2 \geq ab - ac + 2bc$ for all a , b and c .
 - Prove that $2^n > 2n + 1$ if $n \geq 3$.
- There are N boys and N girls in a dance class. How many ways are there to arrange them in pairs to dance?
 - The number $15A$ is divisible by 6. Is A divisible by 6?
- Let a , b and c be positive real numbers. Prove that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

- Solve $|x - 3| + |x + 1| = 4$.
- Prove that no integer in the sequence $11, 111, \dots$ is perfect square.
- How many five digit numbers are there that are the same when the order of their digits is inverted?
- Which of the following fractions is greater? Justify your answer using algebra.

$$\frac{10^{100} + 1}{10^{99} + 1} \text{ and } \frac{10^{99} + 1}{10^{98} + 1}.$$

- The centroid G of $\triangle ABC$, is such that $AG = BC$. Show that $\angle BGC = 90^\circ$.
- Find all pairs (a, b) of real numbers, such that whenever α is a root of $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of the equation.

13. *Two parallel chords in a circle have lengths 10 and 14 and the distance between them is 6. What is a, the length of the parallel chord that is mid way between them?*