

Assam Academy of Mathematics
MATHLETICS – 2015
CATEGORY – III
(Classes IX and X)

Marks : $10 \times 10 = 100$

Time : 3 Hours

(11 a.m. to 2 p.m.)

[Answer in English or in your mother tongue. Two students of the group will discuss the solution of the problems and write the answer in a khata for the group. No third person can help the group to solve the problems]

Answer the following ten questions

1. What is the smallest number in which alternate figures are zero and which is divisible both by 11 and 3 ?
2. Show that $f(n) = n^5 + n^4 + 1$ is not prime for $n > 1$
3. Find all triptets of natural numbers (a, b, c) such that a, b, c are in geometric progression and $a + b + c = 57$.
4. Consider the sequence 2, 3, 5, 6, 7, 8, 10, 11, of all positive integers that are not perfect squares. Find the 2013 th term of the sequence.
5. Suppose x and y are real numbers that satisfy the system of equations

$$2^x - 2^y = 1, \quad 4^x - 4^y = \frac{5}{3}$$

Determine $x - y$.

6. For positive integers a, b, c if 6 divides $a + b + c$ then show that 6 divides $a^3 + b^3 + c^3$ also.
7. Can there be a polynomial $f(x)$ with integer coefficients

(turn over)

so that $f(7) = 11$, $f(11) = 13$.

8. A straight line is drawn through the centre of a square ABCD intersecting side AB at the point N so that $AN : NB = 1 : 2$. On this line take an arbitrary point M lying inside the square. Prove that the distances from M to the sides of the square AB, AD, BC, CD taken in that order form an arithmetic progression.
 9. Which smallest positive integer becomes 57 times smaller by striking its first digit?
 10. Consider three points in cartesian coordinate system given by $O(0,0)$, $A(21,0)$ and $B(0,21)$. Find the number of points in the interior of the triangle OAB having only integer coordinates.
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