Assam Academy of Mathematics

MATHLETICS – 2015

CATEGORY–IV

(Classes XI and XII)

Marks : $10 \times 10 = 100$ **Time : 3 Hours**

(11 a.m. to 2 p.m.)

[Answer in English. Two students of the group will discuss the solution of the problems and then write the answer in a khata for the group. No third person can help the group to solve the problems.]

Answer the following ten questions

1. We call a sequence of m consecutive integers a beautiful sequence if its first term is divisible by 1, 2nd term by 2, ..., $(m-1)$th term by $m-1$ and in addition the last term is divisible by m^2. Does a beautiful sequence exist for $(a) m = 20$ and $(b) m = 11$.

2. For a positive integer n let $S(n)$ denote the sum of its digits and $U(n)$ its unit's digit. Determine all positive integer n with property that

$$n = S(n) + U(n)^2$$

3. Solve the system of equations

$$|x^2 - 2x| + y = 1$$

$$x^2 + |y| = 1$$

4. You have six pieces of papers. You pick one or more of them and cut each of them into six smaller pieces. Now you take one or more of the pieces from the lot and cut each of them into smaller pieces and so on. Prove that you will never have 2013 pieces.

5. Four non coplanar points are given. How many boxes have these points as verices ? [A box is bounded by three pairs of parallel planes.]

(turn over)

6. The powers 2^n and 5^n starts with the same digit d. What is this digit?

7. For any real numbers x, y, z prove that

$$|x|+|y|+|z| \le |x+y-z|+|x-y+z|+|-x+y+z|$$

8. Show that all terms of the sequence
   10001, 100010001, 1000100010001, ..., are composite.

9. If polynomial $f(x,y)$ is antisymmetric if $f(x,y)=-f(y,x)$. Prove that every antisymmetric polynomial $f(x,y)$ has the form $f(x,y)=(x-y)g(x,y)$ where $g(x,y)$ is symmetric.

10. Any four of five circles have a common point. Prove that all five circles have a common point.