

Assam Academy of Mathematics

MATHLETICS - 2014

CATEGORY - IV

(Classes XI and XII)

Marks : 100

Time : 3 Hours

(11 a.m. to 2 p.m.)

Attempt all ten questions. Each question carries 10 marks

1. If a, b, c, d are four positive real numbers, then

Prove that
$$\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} \geq 4$$

2. ABCD is a quadrilateral and P, Q are mid points of CD, AB, AP, DQ meet at X, and BP, CQ meet at Y. Prove that

$$\text{Area of } \triangle ADX + \text{Area of } \triangle BCY = \text{Area of quadrilateral PXOY}$$

3. Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

4. If a, b, c, d are four positive real numbers such that $abcd = 1$,

Prove that $(1+a)(1+b)(1+c)(1+d) \geq 16$.

5. In a quadrilateral ABCD, it is given that AB is parallel to CD and the diagonal AC and BD are perpendicular to each other. Show that

(a) $AD \cdot BC \geq AB \cdot CD$ (b) $AD + BC \geq AB + CD$

6. Let ABCD be a convex quadrilateral in which

$$\angle BAC = 50^\circ, \angle CAD = 60^\circ, \angle CBD = 30^\circ \text{ and } \angle BDC = 25^\circ$$

If E is the point of intersection of AC and BD, find $\angle AEB$.

7. If a, b, c are sides of a triangle, Prove the following inequality :

$$\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a} \geq 3$$

(turn over)

8. If $f(x) = x : \mathbb{R} \rightarrow \mathbb{R}$ is a function satisfying the properties:

$$(i) \quad f(-x) = -f(x)$$

$$(ii) \quad f(x+1) = f(x)+1$$

$$(iii) \quad f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2} \text{ for } x \neq 0$$

Prove that $f(x) = x$ for all real values of x .

9. Show that there do not exist positive integers m and n such that

$$\frac{m}{n} + \frac{n+1}{m} = 4$$

10. Let a, b, c be positive real numbers such that $abc = 1$.

Prove that

$$\left(a - 1 + \frac{1}{b}\right) \left(b - 1 + \frac{1}{c}\right) \left(c - 1 + \frac{1}{a}\right) \leq 1.$$