

Assam Academy of Mathematics
MATHEMATICS OLYMPIAD - 2014

CATEGORY - III
(Classes IX and X)

Marks : 100

Time : 11 am to 2 pm

Each problem carries 10 marks

1. Prove that if p is a prime, then \sqrt{p} is an irrational number.
2. If p is a prime greater than 3 then show that $2p + 1$ and $4p + 1$ can not be primes simultaneously.
3. Find the remainder when $(x+1)^n$ is divided by $(x-1)^3$.
4. Let x, y, z be positive real numbers satisfying $x + y + z = 1$.
Prove that $xy(x+y)^2 + yz(y+z)^2 + zx(z+x)^2 \geq 4xyz$.
5. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function satisfying the properties
 - (i) $f(-x) = -f(x)$
 - (ii) $f(x+1) = f(x) + 1$
 - (iii) $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}$ for $x \neq 0$.

Prove that $f(x) = x$, for all $x \in \mathbb{R}$

6. On the sides BC, CA, AB of a triangle ABC points D, E, F are taken in such a way that

$$\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = 2$$

Show that the area of the triangle determined by the lines AD, BE,

(turn over)

CF is $\frac{1}{7} \times \Delta$, where Δ is the area of $\triangle ABC$

7. A circle cuts the sides of $\triangle ABC$ internally as follows :
BC at D, D'; CA at E, E' and AB at F', F. If AD, BE, CF are concurrent, Prove that AD', BE', CF' are concurrent
 8. Points X, Y are taken on the sides CA, AB of $\triangle ABC$. If BX, CY meet at P and $\frac{AX}{XC} = \frac{BY}{YA} = \frac{1}{2}$, Find the value of the ratio $\frac{BP}{PX}$.
 9. Find the number of 2-digit numbers which are even and have different digits.
 10. $A = \{a_1, a_2, a_3, \dots, a_n\}$ and $B = \{b_1, b_2\}$. Find the number of onto functions that can be defined from A to B.
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