MATHEMATICS OPLMYPIAD
ASSAM ACADEMY OF MATHEMATICS
Category III(class IX and X)

Each of the problems carries 10 marks

1.Prove that if p is a prime, then $\sqrt{p}$ is an irrational number.

2.If p is a prime greater than 3 then show that $2p+1$ and $4p+1$ can not be primes simultaneously.

3.find the remainder when $(x+1)^n$ is devided by $(x-1)^3$.

4.let x, y, z, be positive real numbers satisfying $x+y+z=1$.
prove that $xy(x+y)^2 + yz(y+z)^2 + zx(z+x)^2 \geq 4xyz$.

5.if $f : R \to R$ is a function satisfying the properties
$(i)f(-x) = -f(x), (ii)f(x+1) = f(x)+1, (iii)f(\frac{1}{x}) = \frac{f(x)}{(x^2)}$ for $x \neq 0$.
prove that $f(x) = x$ for all $x \epsilon R$.

6.On sides BC, CA, AB of a triangle ABC points D, E, F are taken in such a way that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = 2$. Show that the area of the triangle determined by the lines AD, BE, CF is $\frac{1}{7} \times \triangle$,where $\triangle$ is the area of $\triangle ABC$.

7.A circle cuts the sides of $\triangle ABC$ internally as follows:
BC at $D, D'; CA$ at $E, E'; AB$ at F', F. If AD, BE, CF are concurrent, prove thatAD', BE', CF' are concurrent.

8.points X, Y are taken on the sides CA, AB of $\triangle ABC$. If BX, CY meet at point p and $\frac{AX}{XC} = \frac{BY}{YA} = \frac{1}{2}$,find the value of the ratio $\frac{BP}{PX}$.

9.Find the number of 2-digit numbers which are even and have different digits.

10.$A = \{a_1, a_2, a_3 ....., a_n\}$ and $B = \{b_1, b_2\}$.Find the number of onto functions that can be defined from from A to B.

***The question paper carries total 100 marks.The exam was held on 07-09-2014(sunday) from 11am to 2pm.**