**Assam Academy of Mathematics**
**MATHEMATICS OLYMPIAD - 2015**

**CATEGORY – IV**
**(Classes XI and XII)**

**Marks : 100** **Time : 11 am to 2 pm**

1. Into how many parts at most is a plane cut by $n$ lines ?

2. Let $n$ be a positive intege which is not divisible by 2 or 5. Prove that there is a multiple of $n$ consisting entirely of ones.

3. If $n$ is an integer and $n > 11$ then prove that $n^2 - 19n + 89$ is not a perfect square.

4. In how many ways can you take an odd number of object from $n$ objects ?

5. If a, b, c are sides of a triangle then show that

$$\frac{3}{2} \leq \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < 2$$

6. $a_0, a_1, a_2, \ldots\ldots, a_{100}$ are given positive integers such that

$a_1 > a_0$, $a_2 = 3a_1 - 2a_0$, $a_3 = 3a_2 - 2a_1, \ldots\ldots,$

$$a_{100} = 3a_{99} - 2a_{98}$$

Prove that $a_{100} > 2^{99}$

7. Let $x_1$, $x_2$ be the roots of $x^2 + ax + bc = 0$ and $x_2$, $x_3$ the roots of $x^2 + bx + ac = 0$ with $ac \neq bc$

Show that $x_1, x_3$ are the roots of the equation $x^2 + cx + ab = 0$

*(turn over)*

8. Find all polynomials $p$ satisfying $p(x+1) = p(x) + 2x + 1$

9. A point O inside a convex quadrilateral is joined to its vertices. Find the area of the quadrilateral with centroids $S_1$, $S_2$, $S_3$, $S_4$ of the respective triangles ABO, BCO, CDO and DAO as vertices.

10. What is the maximum area of a quadrilateral with sides 1, 4, 7, 8 ?

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