General Mathematics
08 October, 2017

Time-3 Hrs   **Answer as many as you can**   **Maximum Marks - 100**

1. What is an irrational number? Prove that $\sqrt{7}$ is irrational.   **(1+2 marks)**

2. (a) State Euclid's Division lemma and Euclid's division algorithm.   **(3 marks)**

   (b) How are they related? Prove one using the other.   **(5 marks)**

3. State the fundamental theorem of arithmetic. Using it, prove that there are infinitely many prime numbers.   **(2+4 marks)**

4. Prove that no number in the sequence $11, 111, 1111, \ldots$ is a perfect square.   **(5 marks)**

5. What is a polynomial? Explain the geometrical meaning of zeroes of a polynomial.   **(1+3 marks)**

6. For the quadratic equation $ax^2 + bx + c = 0$, find an expression for the sum and product of the roots in terms of a, b, c.   **(3 marks)**

7. Without determining the square root, say if 1729314159265352 is a square or not.   **(5 marks)**

8. Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible (that is, in lowest terms) for all natural numbers n.   **(6 marks)**

9. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ have integer coefficients and let z be an integer, then prove that $f(z) = 0$ if and only if z divides a_0. Further prove that if $a_n = 1$, then each rational root of f is an integer.   **(6+4 marks)**

10. What is an arithmetic progression? Find a formula for the nth term of an arithmetic progression. Find a formula for the sum of the first n terms of an arithmetic progression.   **(1+2+2 marks)**

11. Prove that $1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{1}{30} n(n+1)(2n+1)(3n^2 + 3n - 1)$.   **(6 marks)**

12. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.   **(5 marks)**

13. Explain briefly how does one plot points on the coordinate plane.   **(2 marks)**

14. Find the distance between the two points $P(x_1, y_1)$ and $Q(x_2, y_2)$.   **(3 marks)**

15. Prove the section formula.   **(5 marks)**

16. Prove the formula for the area of a triangle. Use this formula to find the special case when the triangle is equilateral.   **(5+2 marks)**

17. In a right triangle ABC, right-angled at B, if $\tan A = 1$, then verify that $2 \sin A \cos A = 1$.**(3 marks)**

18. Find the values of $\tan 45°$, $\sin 30°$ and $\cos 60°$.   **(6 marks)**

19. Find the values of the trigonometric ratios when the angle is 90 degrees.   **(3 marks)**

20. Evaluate $\frac{\sin 18°}{\cos 72°}$. **(2 marks)**

21. Prove for any angle A, $\sin^2 A + \cos^2 A = 1$. What is the value of $1 + \tan^2 A$? **(3+1 marks)**

22. If $\sin x + a \cos x = b$, express $|a \sin x - \cos x|$ in terms of a and b, where a and b are nonnegative real numbers. **(6 marks)**

23. Prove that $\sqrt{\sin^4 x + 4 \cos^2 x} - \sqrt{\cos^4 x + 4 \sin^2 x} = \cos 2x$. **(4 marks)**

24. In triangle ABC, we have $3 \sin A + 4 \cos B = 6$ and $4 \sin B + 3 \cos A = 1$. Prove that the value of the angle C is equal to $30°$. **(6 marks)**

25. Write a short note on any mathematical topic you like. **(3 marks)**