PROBLEMS IN ELEMENTARY NUMBER THEORY–CATEGORY II

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1. *If the number* $17293141519A$ *is divisible by* 33 *then find out all possible values of* A.

2. *Find all pairs of integers* (x, y) *such that* $x^2 - y^2 = 25$.

3. *Without determinig the square root say if* 1729314159265352 *is a square or not.*

4. *Show that any positive odd integer* p *is of the form* $6q + 1$ *or* $6q + 3$ *or* $6q + 5$ *where* $q \in \mathbb{Z}$.

5. *Fow what value of* p *is* $2^{2p} + 7.2^p + 1$ *a perfect square?*

6. *Prove that none of the numbers of the sequence* $11, 111, 1111, \cdots, 111\cdots111, \cdots$ *can be a perfect square.*

7. *If* n *is any natural number, prove that* $n^2 + n - 1$ *is always odd.*

8. *Find the smallest integer* k *which when divided by* $6, 5, 4, 3$ *and* 2 *successively leaves remainder* $5, 4, 3, 2$ *and* 1 *respectively.*

9. *Find the last digit of* 7^{14}.

10. *How many zeroes does* $20!$ *end in?*

11. *Prove that the product of four consecutive integers is always* 1 *less than a perfect square.*

12. *Prove that one of the integers* $a, a + 2$ *and* $a + 4$ *is divisible by* 3.

13. *Show that* $3a^2 - 1$ *is not a perfect square for any integer* a.

14. *Prove that* $n^4 + 4$ *is not a prime number* $\forall n > 1$.

15. *Prove that there is only one pair of non-zero integers whose sum is equal to their product.*

16. *Which four digit number aabb is a square?*

17. *Find the four digit number which, on division by* 131, *yields a remainder of* 112 *and on division by* 132 *yields a remainder of* 98.

18. *A positive whole number* $M < 100$ *is represented in bases* $2, 3$ *and* 5 *notation. It was found that in all the* 3 *cases the last digit is* 1 *while in exactly two out of the three cases the leading digit is* 1. *Find* M.

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