Combinations

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1 Important Results

This section contains various important results and short cut techniques which will be used in solving the problems given in the next section.

1. The number of ways of selecting one or more items from a group of n different items is $2^n - 1$.
2. The number of ways of selecting r items from n identical items is 1.
3. The total number of ways of selecting zero or more items from a group of n identical items is $n + 1$.
4. The total number of selections of some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and the rest are alike of third kind is $(p + 1)(q + 1)(r + 1) - 1$.
5. The total number of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind and n different items is $(p + 1)(q + 1)(r + 1)2^n - 1$.
6. $\binom{n}{r} + \binom{n}{n-1} = \binom{n+1}{r}$.
7. $\binom{n}{r} = \binom{n}{n-r}$.
8. Let n and r be non-negative integers such that $1 \leq r \leq n$ then,
$$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}.$$
9. If $\binom{n}{x} = \binom{n}{y}$ then $x = y$ or $x + y = n$.
10. If n is even, then the greatest value of $\binom{n}{r}$ is $\binom{n}{n/2}$ where $0 \leq r \leq n$.
11. If n is odd, then the greatest value of $\binom{n}{r}$ is $\binom{n}{(n+1)/2}$ or $\binom{n}{(n-1)/2}$ where $0 \leq r \leq n$.
12. The product of r consecutove numbers is divisible by $r!$.

13. If n points are given in a plane, so that no three of them are collinear then the number of straight lines that can be drawn through them is $\binom{n}{2}$, the number of triangles that can be drwan through them is $\binom{n}{3}$, and the number of quadrilaterals that can be drawn through them is $\binom{n}{4}$.

14. If n points are given in a plane, so that k points of which are collinear then the number of straight lines that can be drawn through them is $\binom{n}{2} - \binom{k}{2} + 1$, and the number of triangles that can be drawn through them is $\binom{n}{3} - \binom{k}{3}$.

15. The number of points of intersection of n straight lines is $\binom{n}{2}$.

16. Maximum number of points of intersection of n circles is $2.\binom{n}{2}$.

17. Maximum number of points of intersection of n circles and m lines is $2mn$.

18. Number of handshakes between two persons is $\binom{n}{2}$.

19. Number of diagonals in a n sided polygon is $\binom{n}{2} - n$.

20. Number of triangles which are formed by joining vertices of a regular polygon of n sides is $\binom{n}{3}$.

21. Number of triangles which includes two sides of a regular polygon of n sides is n.

22. Number of triangles which include only one side of a regular polygon of n sides is $n\binom{n-4}{1}$.

23. Number of parallelograms formed by intersecting a set of m parallel lines by a set of n parallel lines is given by $\binom{n}{2}\binom{m}{2}$.

24. The total number of ways of dividing n identical objects among r persons, each of whom, can receive $0, 1, 2, \ldots$ or more objects is $\binom{n+r-1}{r-1}$.

25. The total number of ways of dividing n identical items among r persons each of whom can receive at least one item is $\binom{n-1}{r-1}$.

26. The number of non-negative integral solutions of $x_1 + x_2 + \cdots + x_r = n$ is $\binom{n+r-1}{r-1}$.

27. The number of positive integral solutions of $x_1 + x_2 + \cdots + x_r = n$ is $\binom{n-1}{r-1}$.

28. The total number of selections of r things from n different things when each thing can be repeated unlimited number of times is $\binom{n+r-1}{r}$.

29. The number of ways of distributing n identical balls among k boxes, if each box can hold any number of balls is $\binom{n+k-1}{k-1}$.

30. The number of ways of distributing n identical balls among k different boxes, if none of the box is empty is $\binom{n-1}{k-1}$.

2 Problems

Try to answer the following questions based on the things discussed in the previous section.

1. If $\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$ then what is the least possible value of n?
2. If $\binom{n+1}{3} = 2\binom{n}{2}$, then what is the value of n?
3. What is the value of $\binom{n}{r+1} + \binom{n}{r-1} + 2\binom{n}{r}$?
4. What is the value of $\binom{50}{4} + \sum_{r=1}^{6} \binom{56-r}{3}$?
5. A father with 8 children takes 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. What is the number of times he will go to the garden?
6. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B, besides no three of the lines pass through one point, no line passes through both A and B, and no two are parallel. What is the number of points of intersection?
7. Given five line segments of lengths $2, 3, 4, 5, 6$ units, then what is the number of triangles that can be formed by joining these lines?
8. There are 3, 4 and 5 points marked on the three sides of a traingle. Then what is the number of traingles that can be formed with these points as vertices if none of the marked points are at a vertex of the given triangle?
9. There are 16 points in a plane of which 8 are on one line and 8 are on another line. Then what is the number of triangles that can be formed with the points as vertices?
10. There are 10 points in a plane, out of these 6 are collinear, then what is the number of triangles formed by joining these points?
11. Let T_n denotes the number of triangles, which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$ then what is the value of n?
12. In a plane there are 10 points of which 4 are collinear, then what is the number of quadrilaterals that can be made with these points as vertices?
13. A polygon has 170 diagonals, then how many sides will it have?
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3 Answer Key

The answers of the above questions are given below.

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