INEQUALITIES

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1. *If* $x, y, z \geq 0$ *and* $x + y + z = 1$ *then prove that* $(\frac{1}{x} - 1)(\frac{1}{y} - 1)(\frac{1}{z} - 1) \geq 8$.

2. *If* $a > 0, b > 0, a + b = 1$, *show that* $(a + \frac{1}{a})^2 + (b + \frac{1}{b})^2 \geq 12.5$.

3. *If* a, b, c *are unequal positive numbers in H.P. then prove that* $\frac{a+b}{2a-b} + \frac{c-b}{2c-b} \geq 4$.

4. *Show that* $a^2(1 + b^2) + b^2(1 + c^2) + c^2(1 + a^2) \geq 6abc$.

5. *If* $a, b, c > 0$ *then prove that* $\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} > \frac{9}{28}$ *where* $s = a + b + c$.

6. *Given the perimeter of a triangle, prove that the triangle with the greatest area is equilateral.*

7. *If* $a, b > 1$ *prove that* $log_b a + log_a b \geq 2$.

8. *If* $a, b, c > 0$ *and* $a + b + c = 1$ *prove that* $(1 + \frac{1}{a})(1 + \frac{1}{b})(1 + \frac{1}{c}) \geq 64$.

9. *If* $a, b, c > 0$ *prove that* $(a + b + c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \geq 9$.

10. *Prove that* $(\frac{n+1}{2})^n \geq n!$.

11. *Prove that for* $a_i > 0, i = 1, 2, \ldots, n, a_1 a_2 \ldots a_n = 1$, $(1 + a_1)(1 + a_2) \ldots (1 + a_n) \geq 2^n$.

12. *Prove that* $\sqrt{ab} + \sqrt{cd} \leq \sqrt{(a + d)(b + c)}$.

13. *Prove that if* $a, b, c \in R$, $a^2 + b^2 + c^2 = 1$ *then* $-\frac{1}{2} \leq ab + bc + ca \leq 1$.

14. *Prove that for* $a, b > 0$, *we have* $(ab)^{\frac{n}{n+1}} \leq \frac{a+nb}{n+1}$.

15. *Let* a, b, c *be the sides of a triangle, then prove that* $\frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c} \geq 3$.

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