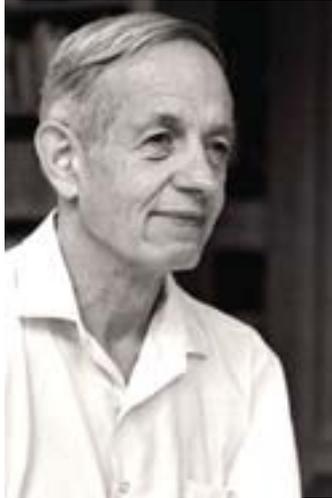


John Forbes Nash Jr – A Short Obituary

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John Forbes Nash, Jr. (June 13, 1928 — May 23, 2015)
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John Forbes Nash, Jr. was born on June 13, 1928 in Bluefield, West Virginia, United States of America. His father, John Forbes Nash, was an electrical engineer for the Appalachian Electric Power Company. His mother, Margaret Virginia Nash, had been a schoolteacher before she married. Nash enrolled at the Carnegie Institute of Technology (now Carnegie Mellon University) first in chemical engineering, he then shifted to chemistry and finally to mathematics in which he graduated in 1948 with a B.S. and a M.S. After his undergraduate studies, he joined Princeton University for his graduate studies and started work on what would later become known as *Nash equilibrium*. After an illustrious career marked by pangs of mental illness, Nash and his wife Alicia Nash died in a car crash on 23 May, 2015.

Nash had said that his interest in mathematics was piqued in some part after he read *Men of Mathematics*, by E T Bell, when he was in high school. As a marker for the great things to come, he completed an important work on bargaining when he was at Carnegie. This work originated as a paper for an elective course in international economics that he had taken, which was perhaps the only economics course that Nash had ever taken.

John Nash is one of those rare mathematicians who are known widely outside academia. This has partly been because of the award winning biography *A Beautiful Mind*, written by Sylvia Nasar [3] and also because of his triumph over a life-threatening mental illness from which he rose and eventually was awarded a long overdue Nobel Memorial Prize in Economic Sciences in 1994.

The most important work for which Nash is widely recognised is his theory of non-cooperative games, which he essentially developed from scratch in his 27-page PhD thesis, [4] submitted to Princeton University in 1950. The thesis, which was written under the supervision of doctoral advisor Albert W. Tucker, contained the definition and properties of the Nash equilibrium, a crucial concept in non-cooperative games. Such games are ones in which players make decisions independently. Thus, while players could cooperate, any cooperation must be self-enforcing. The study of games comes under the purview of a branch of mathematics called *game theory*, which was developed in a strong mathematical footing by the work of von Neumann and Morgenstern in their classic book [7]. The book did not say much about non-cooperative games and later when Nash started publishing his work, both von Neumann and Morgenstern publicly chastised Nash's work as being "just a trivial fixed point theorem".

However, for mathematicians, Nash's work in game theory comes second to his other work in pure mathematics. Many believe that Nash's early career was divided into two parts, the first was at Princeton where he did his pioneering work in game theory and the other was at MIT which he joined in 1951 and where he started working on differential geometry and partial differential equations. While still at graduate school, Nash had completed an important result relating to manifolds and abstract algebraic varieties and he was ready for the eventuality that his work on game theory might not be accepted for a mathematics PhD at Princeton. This work, [5] was

written up in 1951 after he arrived at MIT and the main result is the following theorem.

Theorem 1. *Given any smooth, compact k -dimensional manifold M , there exists a real algebraic variety $V \subset \mathbb{R}^{2k+1}$ and a connected component V_0 of V such that V_0 is a smooth manifold diffeomorphic to M .*

At MIT, he also completed another important work on the isometric problem for Riemannian manifolds. He first did the C^1 embedding, [6] and then attacked the general case. The result that he proved was the following.

Theorem 2. *If $n \leq k(k+1)(3k+11)/2$, then every k -dimensional Riemannian manifold of class C^r can be C^r -isometrically embedded into \mathbb{R}^n , for $r \geq 3$.*

While he was on leave from MIT, at the Institute for Advanced Study in Princeton he started work on another problem involving partial differential equations. Although he succeeded in solving this problem with some novel methods, he was preceded in finding the solution by Ennio de Giorgi of Pisa, Italy. Nash believed that de Giorgi's priority over him in proving this result may have cost him a Fields Medal, considered by many to be the highest honour given to a mathematician under forty years of age.

In 1959, Nash started to show signs of paranoid schizophrenia and gradually started to disappear from academic circles. He spent many years in a precarious condition, only taken care by his wife Alicia, whom he had met while at MIT. During those years, Nash published very little, and most of the scientific community thought him to be dead. But after his return gradually from this state of the mind in the late 1960s, he began to intellectually reject certain delusions that he had. It was a slow recovery to say the least but the support he received from his wife (who had divorced him in the early 1960s and later took him into her house) and colleagues at Princeton enabled him to come out of the ashes.

After much deliberation in 1994, Nash was jointly awarded the Nobel Memorial Prize in Economic Sciences together with John C Harsanyi and Reinhard Setten. With that, he catapulted to almost instant stardom. His biography, [3] and subsequently a Hollywood movie of the same name made Nash the poster child of mathematics in the minds of the general public. After the Nobel, Nash received numerous awards in recogni-

tion of work that he had done more than 40 years ago. He also received financial support to start his research programme and continued to publish a few papers after that. Following his recovery and Nobel, Nash started accepting invitations to lecture all over the world and became a part of academia as he rightly deserved.

His personal life started to take a turn and he remarried Alicia in 2001. In the years following the Nobel prize, Nash worked on a variety of different ideas that he had from game theory and economics to mathematics and physics. In 2015, the Norwegian Academy of Sciences and Letters decided to award the Abel Prize to Nash and Louis Nirenberg "for striking and seminal contributions to the theory of nonlinear partial differential equations and its applications to geometric analysis". This recognition completed a full circle by honouring Nash's early work in mathematics. The Abel Prize, modelled after the Nobel, is considered to be the mathematician's Nobel and it was befitting that Nash should receive this award. Unfortunately, when Nash was returning back after receiving the Abel Prize, he and his wife Alicia succumbed on May 23, 2015 to a car crash in New Jersey. At the time of his death, Nash was 86 years old.

For a more mathematical introduction to Nash's work, readers are referred to [1]. Much of the material for this article was taken from [1] and [2].

References

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