

Regional Mathematical Olympiad-2014

Time: 3 hours

December 07, 2014

Instructions:

- Calculators (in any form) and protractors are not allowed.
- Rulers and compasses are allowed.
- Answer all the questions.
- All questions carry equal marks. Maximum marks: 102.
- Answer to each question should start on a new page. Clearly indicate the question number.

1. In an acute-angled triangle ABC , $\angle ABC$ is the largest angle. The perpendicular bisectors of BC and BA intersect AC at X and Y respectively. Prove that circumcentre of triangle ABC is incentre of triangle BXY .
2. Let x, y, z be positive real numbers. Prove that

$$\frac{y^2 + z^2}{x} + \frac{z^2 + x^2}{y} + \frac{x^2 + y^2}{z} \geq 2(x + y + z).$$

3. Find all pairs of (x, y) of positive integers such that $2x + 7y$ divides $7x + 2y$.
4. For any positive integer $n > 1$, let $P(n)$ denote the largest prime not exceeding n . Let $N(n)$ denote the next prime larger than $P(n)$. (For example $P(10) = 7$ and $N(10) = 11$, while $P(11) = 11$ and $N(11) = 13$.) If $n + 1$ is a prime number, prove that the value of the sum

$$\frac{1}{P(2)N(2)} + \frac{1}{P(3)N(3)} + \frac{1}{P(4)N(4)} + \cdots + \frac{1}{P(n)N(n)} = \frac{n-1}{2n+2}.$$

5. Let ABC be a triangle with $AB > AC$. Let P be a point on the line AB beyond A such that $AP + PC = AB$. Let M be the mid-point of BC and let Q be the point on the side AB such that $CQ \perp AM$. Prove that $BQ = 2AP$.
6. Each square of an $n \times n$ grid is arbitrarily filled with either by 1 or by -1 . Let r_j and c_k denote the product of all numbers in j -th row and k -th column respectively, $1 \leq j, k \leq n$. Prove that

$$\sum_{j=1}^n r_j + \sum_{k=1}^n c_k \neq 0.$$