15 questions on Real Analysis for NET and GATE aspirants

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Find the correct option:

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1. Let $$f:[2,4]\rightarrow R$$ be a continuous function such that $$f(2)=3$$ and $$f(4)=6.$$ The most we can say about the set $$f([2,4])$$ is that

A. It is a set which contains [3,6].

B. It is a closed interval.

C. It is a set which contains 3 and 6.

D. It is a closed interval which contains [3,6].

1. Let $$f:]1,5[\rightarrow R$$ be a continuous function such that $$f(2)=3$$ and $$f(4)=6.$$ The most we can say about the set $$f(]1,5[)$$ is that

A. It is an interval which contains [3,6].

B. It is an open interval which contains [3,6].

C. It is a bounded set which contains [3,6].

D. It is a bounded interval which contains [3,6].

1. Let $$f:]1,5[\rightarrow R$$ be a uniformly continuous function such that $$f(2)=3$$ and $$f(4)=6.$$ The most we can say about the set $$f(]1,5[)$$ is that

A. It is a bounded set which contains [3,6].

B. It is an open interval which contains [3,6].

C. It is a bounded interval which contains [3,6].

D. It is an open bounded interval which contains [3,6].

1. Let $$A$$ be a set. What does it mean for $$A$$ to be finite?

A. is a proper subset of the natural numbers.

B. There exists a natural number n and a bijection f from $\{i\in N:i<n\}$ to $A.$

C. There is a bijection from A to a proper subset of the natural numbers.

D. There exists a natural number n and a bijection f from $\{i\in N:i\leq n\}$ to $A.$

1. Let A be a set. What does it mean for A to be countable?

A. One can assign a different element of A to each natural number in $N.$

B. There is a way to assign a natural number to every element of $A,$ such that each natural number is assigned to exactly one element of $A.$

C. A is of the form $\{a_1,a_2,a_3,\dots\}$ for some sequence $a_1,a_2,a_3,\dots$

D. One can assign a different natural number to each element of $A.$

1. Let A be a set. What does it mean for A to be uncountable?

A. There is no way to assign a distinct element of A to each natural number.

B. There exist elements of A which cannot be assigned to any natural number at all.

C. There is no way to assign a distinct natural number to each element of $A.$

D. There is a bijection f from A to the real numbers $R.$

1. A and B be bounded non-empty sets. Following are two groups of statements:

(i) $\inf(A)\leq \inf(B)$

(ii) $\inf(A)\leq \sup(B)$

(iii) $\sup(A)\leq \inf(B)$

(iv) $\sup(A)\leq \sup(B)$

(p) For every $\epsilon >0$ $\exists a\in A$ & $b\in B$ s.t. $a<b+\epsilon.$

(q) For every $b\in B$ and $\epsilon >0$ $\exists a\in A$ s.t. $a<b+\epsilon.$

(r) For every $a\in A$ and $\epsilon >0$ $\exists b\in B$ s.t. $a<b+\epsilon.$

(s) For every $a\in A$ and $b\in B,$ $a\leq b.$

Find the correct option from the following:

A. $$(i)\Rightarrow (p), (ii)\Rightarrow (s), (iii)\Rightarrow (q), (iv)\Rightarrow (r).$$

B. $$(i)\Rightarrow (q), (ii)\Rightarrow (r), (iii)\Rightarrow (p), (iv)\Rightarrow (s).$$

C. $$(i)\Rightarrow (q), (ii)\Rightarrow (p), (iii)\Rightarrow (s), (iv)\Rightarrow (r).$$

D. $$(i)\Rightarrow (s), (ii)\Rightarrow (q), (iii)\Rightarrow (r), (iv)\Rightarrow (s).$$

1. The radius of convergence of the power series $$\sum a_nx^n$$ is $$R$$ and $$k$$ be a positive integer. Then the radius of convergent of the power series $$\sum a_nx^{kn}$$ is

A. $$\frac{R}{k}.$$

B. $$R.$$

C. not depend on $$k.$$

D. $$R^{\frac{1}{k}}.$$

1. Let $$f:R\rightarrow R$$ s.t. $$f(x)=\left{\begin{array}{ll} \frac{|x|}{x} & \mbox{if $$x\neq0$$}; 0 & \mbox{if $$x=0$$}. \end{array} \right}.$$

And

$$g:R\rightarrow R$$ s.t. $$g(x)=\left{\begin{array}{ll}

\frac{|x|}{x} & \mbox{if $$x\neq0$$};

1 & \mbox{if $$x=0$$}.

\end{array} \right}.$$

Then,

A. f and g both are continuous at x=0.

B. Neither f nor g is continuous at x=0.

C. f is continuous at x=0, but g is not.

D. g is continuous at x=0, but f is not.

1. Let $$f(x)=\left{\begin{array}{ll}

8x & \mbox{for $$x\in Q$$};

2x^2+8 & \mbox{for $$x\in Q^c$$}.

\end{array} \right}.$$

Then,

A. f is not continuous.

B. f is continuous at x=0.

C. f is continuous at x=2.

D. f is continuous at both x=0 and x=2.

1. $$f(x)=\left{\begin{array}{ll} X^2-1 & \mbox{if $$x\in Q$$}; 0 & \mbox{if $$x\in Q^c$$}. \end{array} \right}.$$

Then,

A. f is not continuous.

B. f is continuous at x=1, but not continuous at x=-1.

C. f is continuous at both x=1 and x=-1.

D. f is continuous at x=-1, but not continuous at x=1.

1. Let $$f:\Rightarrow R$$ be continuous and $$f(x)=\sqrt{2} \forall x\in Q.$$ Then $$f(\sqrt{2})$$ equals to

A. $$\sqrt{2}.$$

B. 0.

C. Neither $$\sqrt{2}$$ nor $$0.$$

D. None of these.

1. Let $$f:\Rightarrow R$$ be continuous, $$f(0)<0$$ and $$f(1)>1.$$ Then,

(i) There exist $$c\in (0,1)$$ such that $$f(c)=c^2.$$

(ii) There exist $$d\in (0,1)$$ such that f(d)=d.

A. (i) is true, but (ii) is not true.

B. (ii) is true, but (i) is not true.

C. Both (i) and (ii) are true.

D. None of above.

1. $$f:\Rightarrow {-1,1}$$ be onto. Then

A. f is not continuous.

B. f is continuous.

C. f is differentiable everywhere.

D. f is continuous, but not differentiable anywhere.

1. The sequence $${\frac{\sin{\frac{n\pi}{2}}}{n}}_{n=1}^{\infty}$$

A. is convergent.

B. is divergent.

C. converges to 0.

D. converges to 1.

The answers are here.
