50 questions on linear algebra for NET and GATE aspirants

by Gonit Sora - Friday, June 22, 2012

https://gonitsora.com/50-questions-linear-algebra-net-gate-aspirants/

Find the correct options:

1) $M=\left(\begin{array}{ccc}1 & 2 & 2 \\0 & 2 & 2 \\0 & 1 & 1 \end{array}\right)$ and $V=\{ Mx^{T} : x\in R^{3}\}.$ Then $dim V$ is

(a) 0 (b) 1 (c) 2 (d) 3

2) $A^{2}-A=0,$ where A is a 9×9 matrix. Then

(a) A must be a zero matrix (b) A is an identity matrix

(c) rank of A is 1 or 0 (d) A is diagonalizable

3) The number of linearly independent eigen vectors of $\left(\begin{array}{cccc}1 & 1 & 0 & 0 \\2 & 2 & 0 & 0 \\0 & 0 & 3 & 0\\0 & 0 & 5 & 5 \end{array}\right)$ is

(a) 1 (b) 2 (c) 3 (d) 4

4) The minimal polynomial of $\left(\begin{array}{cccc}2 & 1 & 0 & 0 \\0 & 2 & 0 & 0 \\0 & 0 & 2 & 0\\0 & 0 & 0 & 5 \end{array}\right)$ is

(a) $(x-2)$ (b) $(x-2)(x-5)$ (c) $(x-2)^{2}(x-5)$ (d) $(x-2)^{3}(x-5)$

5) A is a unitary matrix. Then eigen value of A are

(a) 1, -1 (b) 1, -i (c) i, -i (d) -1, i

6) $latex \left(\begin{array}{ccc}2 & -3 \\2 & -2 \end{array}\right)$ is an operator on $latex R^{2}.$ The invariant subspaces of the operator are

(a) $latex R^{2}$ and the subspace with base {(0,1)} (b) $latex R^2$ and the zero subspace

(c) $latex R^2,$ the zero subspace and the subspace with base {(1,1)} (d) only $latex R^{2}$

7) Rank of the matrix $latex \left(\begin{array}{ccccc}21 & -7 & 0 & 0 & 0 \\-11 & 9 & 0 & 0 & 0 \\0 & -19 & 35 & 0 & 0 \\0 & 15 & 0 & 12 & 20 \\0 & 0 & -24 & 21 & 35 \end{array}\right)$ is

(a) 2 (b) 3 (c) 4   (d)   5

8) The dimension of the subspace of $latex M_{2\times 2}$ spanned by $latex \left(\begin{array}{ccc}1 & -5 \\-4 & 2 \end{array}\right),$ $latex \left(\begin{array}{ccc}1 & 1 \\-1 & 5 \end{array}\right)$ and $latex \left(\begin{array}{ccc}2 & -4 \\-5 & 7 \end{array}\right)$ is

(a) 1 (b) 2 (c) 3   (d)   4

9) U and V are subspace of $latex R^{4}$ such that

U = span [(1,2,3,4), (5,7,2,1), (3,1,4,-3)]

V=span [(2,1,2,3), (3,0,1,2), (1,1,5,3)].

Then the dimension of $latex U\cap V$ is

(a) 1 (b) 2 (c) 3   (d)   4

10) Let $latex M_{n\times n}$ be the set of all n-square symmetric matrices and the characteristics polynomial of each $latex A\in M_{n\times n}$ is of the form

$latex t^{n}+t^{n-2}+a_{n-3}t^{n-3}+\dots +a_{1}t+a_{0}.$ Then the dimension of $latex M_{n\times n}$ over R is

(a) $latex \frac{(n-1)n}{2}$ (b) $latex \frac{(n-2)n}{2}$ (c) $latex \frac{(n-1)(n+2)}{2}$ (d) $latex \frac{(n-1)^{2}}{2}$

11) A is a $latex 3\times 3$ matrix with $latex \sigma (A)={1, -1, 0 }.$ Then $latex |I+A^{100}|$ is

(a) 6 (b) 4 (c) 9 (d) 100

12) A is $latex 5\times 5$ matrix, all of whose entries are 1, then

(a) A is not diagonalizable (b) A is idempotent (c) A is nilpotent

(d) The minimal polynomial and the characteristics polynomial of A are not equal.

13) A is an upper triangular with all diagonal entries zero, then I+A is

(a) invertible (b) idempotent (c) singular (d) nilpotent

14) Number of linearly independent eigen vectors of $latex \left(\begin{array}{cccc}2 & 2 & 0 & 0 \\2 & 1 & 0 & 0 \\0 & 0 & 3 & 0 \\0 & 0 & 1 & 4 \end{array}\right)$ is

(a) 1 (b) 2 (c) 3 (d) 4

15) A is a $latex 5\times 5$ matrix over $latex R,$ then $latex (t^{2}+1)(t^{2}+2)$

(a) is a minimal polynomial (b) is a characteristics polynomial

(c) both (a) and (b) are true (d) none of (a) and (b) is true

16) M is a 2-square matrix of rank 1, then M is

(a) diagonalizable and non singular (b) diagonalizable and nilpotent

(c) neither diagonalizable nor nilpotent (d) either diagonalizable or nilpotent

17) A be a n-square matrix with integer entries and $latex B=A+\frac{1}{2} I.$ Then

(a) B is idempotent (b) $latex B^{-1}$ exist (c) B is nilpotent (d) B-I is idempotent

18) Let $A\in M_{3\times 3}(R),$ then $t^{2}+1$ is

(a) a minimal polynomial of A (b) a characteristics polynomial of A

(c) both (a) and (b) are true (d) none of (a) and (b) is true

19) A is a 4-square matrix and $A^{5}=0.$ Then

(a) $A^{4}=I$ (b) $A^{4}=A$ (c) $A^{4}=0$ (d) $A^{4}=-I$

20) $A=\left(\begin{array}{ccc}0 & 1 & a \\0 & 0 & 1 \\0 & 0 & 0 \end{array}\right)$ and $B=\left(\begin{array}{ccc}0 & 1 & 0 \\0 & 0 & 1 \\0 & 0 & 0 \end{array}\right).$ Then

(a) A and B are similar (b) A and B are not similar

(c) A and B are nilpotent (d) A and AB are similar

21) Let $S=\{ 2-x+3x^{2}, x+x^{2}, 1-2x^{2} \}$ be subset of $P_{2}(R).$ Then

(a) S is linearly independent (b) S is linearly dependent

(c) (2,-1,3), (0,1,1), (1,0,-2) are linearly dependent (d) S is a basis of $P_{2}(R)$

22) $T: P_{2}(R)\rightarrow P_{3}(R)$ such that $T(f(x))=2f^{ '}(x)+3 \int_{0}^{x}f(t)dt.$ Then rank of T is

(a) 1 (b) 2 (c) 3 (d) 4

23) $T_{i}: P(R)\rightarrow P(R)$ such that $T_{1}(f(x))= \int_{0}^{x}f(t)dt$ and $T_{2}(f(x))=f^{'}(x).$ Then

(a) T_{1} is 1-1, T_{2} is not (b) T_{2} is 1-1, T_{1} is not

(c) T_{1} is onto and T_{2} is 1-1 (d) T_{1} and T_{2} both are 1-1

24) $latex T: P_{3}(R)\rightarrow M_{2\times 2}(R),$ such that $latex T(f(x))= \left(\begin{array}{ccc}f(1) & f(2)\\f(3) & f(4)\end{array}\right),$ then

(a) T is 1-1 (b) T is onto (c) T is both 1-1 and onto (d) T is neither 1-1 nor onto

25) W=L{(1,0,0,0), (0,1,0,0)}, then

(a) $latex \frac{R^{4}}{W}=L{ W+(2,0,0,0), W+(0,2,0,0) }$

(b) $latex \frac{R^{4}}{W}=L{ W+(1,2,3,4), W+(2,3,4,5) }$

(c) $latex \frac{R^{4}}{W}=L{ W+(0,0,2,0), W+(0,0,0,2) }$

(d) $latex { W+(1,2,3,4), W+(2,3,4,5) }$ is a basis of $latex \frac{R^{4}}{W}$

26) $latex A= \left(\begin{array}{ccc}0 & 1 \\0 & 0 \end{array}\right),$ then

(a) A has zero image (b) all the eigen value of A are zero

(c) A is idempotent (d) A is nilpotent

27) $latex T:R^{4}\rightarrow R^{4},$ defined by $latex T(e_{1})=e_{2}, T(e_{2})=e_{3}, T(e_{3})=0, T(e_{4})=e_{3}.$ Then

(a) T is nilpotent (b) T has at least one non-zero eigen value

(c) index of nilpotent is three (d) T is not nilpotent

28) $latex A= \left(\begin{array}{ccc}a & a & a \\a & a & a \\a & a & a \end{array}\right),$ where $latex a\neq 0,$ then

(a) A is not diagonalizable (b) A is idempotent

(c) A is nilpotent (d) minimal polynomial ? characteristics polynomial

29) $latex A\in M_{2\times 2}(R)$ and rank of A is 1, then

(a) A is diagonalizable (b) A is nilpotent

(c) both (a) and (b) are true   (d)   none of (a) and (b) is true

30) A is a 3-square matrix and the eigen values of A are -1, 0, 1 with respect to the eigen vectors $latex (1,-1,0)^{T}, (1,1,-2)^{T}, (1,1,1)^{T}.$ then 6A is

(a) $latex \left(\begin{array}{ccc}1 & 5 & 3 \\5 & 1 & 3 \\3 & 3 & 3 \end{array}\right)$

(b) $latex \left(\begin{array}{ccc}-1 & 5 & 2 \\5 & -1 & 2 \\2 & 2 & 2 \end{array}\right)$

(c) $latex \left(\begin{array}{ccc}1 & 1 & 1 \\-1 & 1 & 1 \\0 & -2 & 1 \end{array}\right)$

(d) $latex \left(\begin{array}{ccc}6 & -1 & 0 \\1 & 6 & -2 \\1 & 1 & 6 \end{array}\right)$

31) The sum of eigen values of $latex \left(\begin{array}{ccc}-1 & -2 & -1 \\-2 & 3 & 2 \\-1 & 2 & -3 \end{array}\right)$ is

(a) -3 (b) -1 (c)   3 (d)   1

32) The matrix $latex \left(\begin{array}{ccc}a^{2} & ab & ac \\ab & b^{2} & bc \\ac & bc & c^{2} \end{array}\right),$ where $latex a,b,c\in R-{ 0}$ has

(a) three real, non-zero eigen values (b) complex eigen values

(c) two non-zero eigen values   (d) only one non-zero eigen value

33) $latex \left(\begin{array}{ccc}2 & 0 & 0 \\1 & 2 & 0 \\0 & 0 & 1 \end{array}\right)$ is

(a) diagonalizable (b) nilpotent (c) idempotent   (d) not diagonalizable

34) If a square matrix of order 10 has exactly 5 distinct eigen values, then the degree of the minimal polynomial is

(a) at least 5 (b) at most 5 (c) always 5 (d) exactly 10

35) $latex T: R^{2\times 2}\rightarrow R^{2\times 2},$ defined by T(A)=BA, where $latex B=\left(\begin{array}{ccc}1 & -2 \\-2 & 4 \end{array}\right).$ Then rank of T is

(a) 1 (b) 2 (c) 3 (d) 4

36) $latex A=\left(\begin{array}{ccc}1 & a & b \\0 & 10 & c \\0 & 0 & 100 \end{array}\right).$ Then

(a) both $latex A$ and $latex A^{2}$ are diagonalizable (b) $latex A$ is diagonalizable but not $latex A^{2}$

(c) $latex A$ and $latex A^{2}$ have the same minimal polynomial (d) $latex A^{2}$ is diagonalizable but not $latex A$

37) Rank of $latex A_{7\times 5}$ is 5 and that of $latex B_{5\times 7}$ is 3, then rank of AB is

(a) 1 (b) 2 (c) 3 (d) 4

38) A and B are n-square positive definite matrices. Then which of the following are positive definite.

(a) A+B (b) ABA (c) AB (d) $latex A^{2}+I$

39) $latex A\in M_{3\times 3}(R)$ and $latex A=\left(\begin{array}{ccc}2 & 1 & 0 \\0 & 2 & 0 \\0 & 0 & 3 \end{array}\right),$ then which of the following are subspaces of $latex M_{3\times 3}(R)$

(a) $latex {X\in M_{3\times 3}(R): XA=AX}$ (b) $latex {X\in M_{3\times 3}(R):X+A=A+X}$

(c) $latex {X\in M_{3\times 3}(R): trace(AX)=0}$ (d) $latex {X\in M_{3\times 3}(R): det(AX)=0}$

40) Let T be a linear operator on the vector space V and T be invariant under the subspace W of V. Then

(a) $latex T(W)\in W$ (b) $latex W\in T(W)$ (c) $latex T(W)=W$ (d) None of these

41) $latex A : R^{4}\rightarrow R^{3},$ where $latex A=\left(\begin{array}{cccc}1 & 2 & 3 & 1 \\1 & 3 & 5 & -2\\3 & 8 & 13 & -3 \end{array}\right).$ Then the dimension of kernel of A is

(a) 1 (b) 2 (c) 3 (d) 4

42) $latex A : R^{3}\rightarrow R^{4},$ where $latex A=\left(\begin{array}{ccc}1 & 1 & 3 \\2 & 3 & 8 \\3 & 5 & 13 \\1 & -2 & -3 \end{array}\right).$ Then the dimension of image of A is

(a) 1 (b) 2 (c) 3 (d) 4

43) Let u, v, w be three non-zero vectors which are linearly independent, then

(a) u is linear combination of v and w (b) v is linear combination of u and w

(c) w is linear combination of u and v (d) none of these

44) Let U and W be subspaces of a vector space V and $latex U\cup W$ is also a subspace of V, then

(a) either $latex U\subseteq W$ or $latex W\subseteq U$ (b) $latex U\cap W= \phi$ (c) U=W (d) None of these

45) Let I be the identity transformation of the finite dimensional vector space V, then the nullity of I is

(a) dimV (b) 0 (c) 1 (d) dimV - 1

46) $latex T : R^{3}\rightarrow R^{3}$ such that $latex T(a,b,c)=(0, a,b),$ for $latex (a,b,c)\in R^{3}.$ Then $latex T+I$ is a zero of the polynomial:

(a) $latex t$ (b) $latex t^{2}$ (c) $latex t^{3}$ (d) none of above

47) The sum of the eigen values of the matrix $latex \left(\begin{array}{ccc}4 & 7 & 11 \\7 & 1 & -21\\11 & -21 & 6\end{array}\right)$ is

(a) 4 (b) 23 (c) 11 (d) 12

48) Let A and B are square matrices such that AB=I, then zero is an eigen value of

(a) A but not of B (b) B but not of A (c) both A and B (d) neither A nor B

49) The eigen values of a skew-symmetric matrix are

(a) negative (b) real (c) absolute value of 1 (d) purely imaginary or zero

50) The characteristics equation of a matrix A is $latex t^{2}-t-1=0,$ then

(a) $latex A^{-1}$ does not exist (b) $latex A^{-1}$ exit but cannot be determined from the data

(c) $latex A^{-1}=A+1$ (d) $latex A^{-1}=A-1$

The answers can be found here.