A Brief History of Ancient Indian Mathematics (part I)

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(India's Romance with Numbers)

It is without doubt that mathematics, the science of numbers, today owes a huge debt to the outstanding contributions made by Indian mathematicians over many hundreds of years. From zero to geometry, Indian mathematicians made some great historical achievements. Frankly speaking, without the Indian numerals, mathematics, as we know it today, would simply not exist. This article is to focus on some of these achievements and to show that mathematics was not only very much rooted in Indian soil but also that ancient Indians had great romance with mathematics.

Baudhayana (800 BC)

Indian mathematics may be said to have started with the Vedic rituals which required knowledge of geometry for accurate construction of Vedic altars. It developed further under the Jain and the Buddhist scholars who pioneered some phenomenal ground level achievements. It is now generally admitted that the Indian system of numbers has its roots firmly planted in India and that it is the Indians who first invented and used the decimal place value system including the use of the zero. The novel Indian numerals were subsequently adopted by the Arabs, and eventually became known to Europe as Arabic numerals. The ancient Indians provided a unique, useful, flexible and intuitive model for the world to use.

The great French Mathematician, Pascal was one who appreciated the contributions of the Indians and he put this with great clarity, when he commented, "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius."

India has a long tradition, both historical and mythical, of its fascination with numbers. In ancient India, mathematics was considered as one of the highest sciences. There is a statement in the Vedanga Jyotisa, which proclaims, "As are the crests of a peacock, as are the gem-stones of a snake, placed on the highest place of the body, the forehead, so is mathematics (Ganita) the head of all Vedah and shastras." The quotation suggests a reverent, almost elitist concept of mathematics in ancient India. In another mythical statement we find, "What is the use of much speaking. Whatever object exists in this moving and nonmoving world, cannot be understood without the base of Ganita (Mathematics)". Three thousand years later, Gaileo would realise the same when he said, "It (the universe) is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word; without these, one is wandering about in a dark labyrinth."

The tradition of mathematics in India, in fact, started much earlier of which we have solid historical facts in the Indus valley which was associated with the Harappan civilization established around 2,500 B.C. We do know that the Harappans had adopted a uniform system of weights and measures. An analysis of the weights discovered suggests that they are decimal in nature, giving for the main series ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500. Several scales for the measurement of length were also discovered during excavations. One was a decimal scale based on a unit of measurement of 1.32 inches (3.35 centimeters) which has been called the "Indus inch". Of course ten units is then 13.2 inches which is quite believable as the measure of a "foot". Another scale was discovered when a bronze rod was found which was marked in lengths of 0.367 inches. It is surprising to see the accuracy with which these scales are marked. Now 100 units of this measure is 36.7 inches which is the measure of a stride. Measurements of the ruins of the buildings which have been excavated show that these units of length were accurately used by the Harappans in construction.

It is not known fully if the Harappan knowledge in mathematics was continued in later Vedic period. However we do know that from ancient time, Indian minds fascinated with higher numbers. While the Greeks had no terminology for denominations above a myriad $latex (10^{4})$ and the Romans above Millie $latex (10^{3}),$ ancient Indians dealt freely with no less than eighteen denominations. We know from a record of an early Buddhist work in 5th century BC, how the prince Gautama Buddha correctly recited the counting beyond the koti on the centesimal scale: “Hundred kotis are called ayuta, hundred ayutas niyuata, hundred niyutas kankara, hundred kankaras vivara, hundred vivaras ksobhya, hundred ksobhyas vivaha, hundred vivahas ustanga, hundred ustangas babula, hundred babulas nagabala, hundred nagabalas tithi lambha and so on upto another twelve terms ending with tallaksana. (Thus one Tallaksna is $latex 10^{53}$).

Aryabhatta

Examples of geometric knowledge (rekha-ganit) are to be found in the Sulva-Sutras of Baudhayana (800 BC) and Apasthmaba (400 BC). The term Sulvasutra means “the rules of the chord”; it is the name given to the supplements of the Kalpasutras which explain the construction of sacrificial Vedic altars. A statement in Baudhayana's Sulvasutra runs,”In a Deerghchatursh (Rectangle) the Chetra (Square) of Rajju (hypotenuse) is equal to sum of squares of Parshvamani (base) and Triyangmani (perpendicular).” This is what is commonly known as the Pythagoras theorem. This shows that Vedic Indians had knowledge of the Pythagoras theorem as early as 8th century B.C. Apasthamba's sutra provides a value for the square root of 2 that is accurate to the fifth decimal place. Apasthamba also looked at the problems of squaring a circle, dividing a segment into seven equal parts, and a solution to the general linear equation.

However we do not find any proof of any of the theorems in Vedic mathematics, just the statements. This may be because, the involved mathematics was considered a sacred and secret knowledge just for the Vedic rituals reserved for the priests only. This has been one of the main problems for further development on Indian science during the Vedic Brahmanic age. As a result, none of these geometrical constructions appeared in any subsequent Indian literature, and later mathematicians did not carry these discussions to any higher level like what the Greeks did. As Gordon Childe, the famous historian rightly puts it: while the Greeks were free to speculate on „facts of common experience? and „the practice of the craft?, the Vedic Indians were restricted by their „inheriting from the Bronze Age the sacred hymns of Veda and ritual manuals verbally remembered?. In fact the Indians could never got rid of the Vedas completely which prevented them, throughout the ages, from exploring „secular? scientific speculation to higher limits.

Thus it was mainly under the religious and philosophical impulses of the Jains and the Buddhists, that Indian mind learned to speculate outside the Vedas, and science and mathematics got some freedom. In ancient India, mathematics was a very lively passion. In Sanskrit, Ganita literally means the science of calculations which were generally done on a board (pati) with a piece of chalk or on sand (dhuli) spread on the ground. Thus the terms Pati-Ganita (science of calculations on the borad) and Dhuli-Karma (dust work) came to be used for higher mathematics. In ancient Buddhist literature we find mention of three classes of Ganita (1) Mudra – finger arithmetic (2) Ganana – mental arithmetic (3) Samkhyana – higher mathematics in general.

These Jains and Buddhist scholars worked on problems such as number theory, cubic equations, quadratic equations, and statistics. They also had an understanding of advanced ideas such as that of infinity. The Jains were the first to discard the idea that all infinites were the same or equal. They recognized four different types of infinities: infinite in length (one dimension), infinite in area (two dimensions), infinite in volume (three dimensions), and infinite perpetually (infinite number of dimensions). Jain texts from the 6th C BC such as the Surya Pragyapti describe ellipses. Buddhist literature also demonstrates an awareness of indeterminate and infinite numbers. Numbers were deemed to be of three types: Sankheya (countable), Asankheya (uncountable) and Anant (infinite). The Buddhist philosophical formulations concerning Shunya - i.e. emptiness or the void may have facilitated in the introduction of the concept of zero. While the zero (bindu) as an empty place holder in the place-value numeral system appears much earlier, algebraic definitions of the zero and its relationship to mathematical functions appear in the mathematical treatises of Brahmagupta in the 7^{th} C AD.

The early Jainas seem to have great liking for the subject of combinations and permutations. Mahabira, the founder of Jainism, was himself a great mathematician. In the Bhagawati sutra are set forth simple problems such as finding the number of combinations that can be obtained from a given number of fundamental philosophical categories taken one at a time, two at a time, and three at a time or more at a time. The Jaina commentator Silanka has quoted three rules regarding permutations and combinations. The Jains were the first to conceive of transfinite numbers, a concept, which was brought to Europe by Cantor in the late 19th century. The two thousand year old Jaina literature may hold valuable clues to the very nature of mathematics. This is one area where further research could prove very fruitful.

The works of the early Jain and Buddhist scholars were later summarized and expanded by Aryabhatta (476-550), the most important ancient mathematician of India. Aryabhatta headed the classical era of Indian mathematics. He helped to ignite a new era in mathematics, which in turn spurred on other sciences, such as astronomy. Among his many accomplishments were the introduction of the concept of trigonometry, the most precise estimation of ? (the ratio of the circumference to the diameter of a circle) up to that date (3.1416), and an accurate estimation of a solar year. His calculations on the circumference of the earth (62832 miles) and the length of the solar year (within about 13 minutes of the modern calculation) were remarkably close approximations. In making such calculations, Aryabhatta had to solve several mathematical problems that had not been addressed before including problems in algebra (bij-ganit) and trigonometry. In the course of developing a precise mapping of the lunar eclipse, Aryabhatta was obliged to introduce the concept of infinitesimals - i.e. tatkalika gati to designate the infinitesimal, or near instantaneous motion of the moon, and express it in the form of a basic differential equation. It is worth mentioning that Roots of the Modern Trignometry lie in the book titled Surya Siddhanta . It mentions Zia (Sine), Kotizia (Cosine) etc. Please note that the word (Zia) changed to "Jaib" in Arab first. The translation of Jaib in Latin was done as "Sinus". And this "Sinus" became "Sine" later on. The word Trigonometry also is derived from the Indian word Trikonomiti, Trikono meaning a Triangle, (modern day Tribhuja).

A Brief History of Ancient Indian Mathematics (part II)?

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