

Anti-Dühring: 1+2+3+4+...

by Harun Šiljak - Wednesday, November 25, 2015

<http://gonitsora.com/anti-duhring-1234/>

Anti-Dühring is a book by Friedrich Engels published in 1878. It is a defense of Marxism from Eugen Dühring's vision of socialism. It contains interesting scientific references as well, which is the reason I am putting a spotlight on it.

First of all, it would be polite to explain: why is this series of columns called antifeuilleton?

Idea is simple: inherit the good sides of feuilleton and at the same time, get as far as possible from the bad sides of it as a written form (the manner of expression, making opinions) Hermann Hesse criticized in *Das Glasperlenspiel*. When Hesse writes about the age of feuilletons, feuilletons like “Friedrich Nietzsche and Women’s Fashions of 1870”, he targets at the banalization and the environment in which in his vision, Castalia as the Knowledge State is born. Castalia wouldn’t be itself in Hesse’s work if there wasn’t for the glass beads game – art of merging different scientific and artistic forms with a common motif.

Antifeuilleton is an attempt of a homage for Hesse’s vision: running away from triviality of feuilleton using the basic idea of the glass beads game: magical intertwining of two seemingly unrelated stories from science, art, history. The reader will be the judge of this attempt’s success.

First motif came naturally from Anti-Dühring. It is not just the same prefix (Anti-Dühring, antifeuilleton), though: Friedrich Engels published Anti-Dühring in a newspaper as a serial. The motif lies in the fifth chapter of part one – Philosophy of nature, time and space, where two paragraphs are vital for our story. First one is: “Herr Dühring, on the other hand, compels us to accept six dimensions in space, and immediately afterwards can find no words to express his indignation at the mathematical mysticism of Gauss, who would not rest content with the usual three dimensions of space.” The second one is just a few paragraphs below, when Engels mentions the same series as Dühring in his own argumentation: $1+2+3+4+...$ when arguing about the (in)finiteness of space and time.

Most readers will say that $1+2+3+...$ is simple as one, two, three: it’s an infinite series and no discussion can be made there. Leonhard Euler was one of the people who proved $1+2+3+...=-1/12$, observing it as a curiosity and abuse of mathematical rules. Positive integers adding up to a negative fraction? Oh, please. Ramanujan wrote to Hardy about it: “I told him that the sum of an infinite no. of terms of the series $1+2+3+4+...=-1/12$ under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal.”

Don’t worry, [Ramanujan](#) was no lunatic. Neither was [Euler](#), nor [Abel](#), who was also aware of the possibility of summing divergent series and obtaining these seemingly absurd results. Abel tried to stay rational: “The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever.” This was the word of supreme wisdom for years, but the card of divergent series found its place in the solitaire of modern science one day – and it jumped straight into the realm of physics. This will be the topic of the next story, Occupation in 26 Dimensions.

At this point, I owe you an explanation what the cryptic (and so obviously wrong) equality $1+2+3+4+\dots=-1/12$ might mean. I will use the notorious feuilletonistic freedom to stay superficial.

It shouldn't be too restrictive to assume my reader has heard of the [Millenium problems](#), the set of open mathematical questions with a bounty offered by Clay Institute (although it has been shown by an explicit example that the bounty isn't primarily inspiring brilliant mathematicians). One of these problems is the [Riemann Hypothesis](#). I won't write about the hypothesis itself, but I'll take the basic ingredient of it, the Riemann zeta function, defined as $\zeta(s)=1^{-s}+2^{-s}+3^{-s}+\dots$, converging for s with real part larger than one. Zeta function can also be defined using a functional equation $F(s)=F(1-s)$ where $F(s)=\pi^{-s/2}\Gamma(s/2)\zeta(s)$ (if this is too many Greek letters for you, you can safely skip this whole paragraph) with the usual definition of function branches. Using $\Gamma(1/2)=\sqrt{\pi}$ and $\zeta(2)=\pi^2/6$ we get $\zeta(-1)=1/12$. Of course, we decided to ignore the caveat that the real part of s has to be larger than one.

Why all this and what does it have to do with Düring's 6 dimensions? Wait for the next move in glass beads game.

[This is the first of a 10 article series to be written by the author in the coming few weeks for Gonit Sora.]

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