Applications of Lie Groups to Difference Equations

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http://gonitsora.com/applications-lie-groups-difference-equations/

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Chapman & Hall/ CRC, 2010, 344 pp.

The invariance of differential equations under transformation groups, usually Lie groups, naturally comes from the models they formalise, and there is a lot of literature dedicated to applications of Lie groups in differential equations. The difference schemes for solving differential equations, on the other hand, discretise the space, and it appears to be important that such a scheme preserves the symmetries of the original model. This book is dedicated to the study of such symmetry-preserving difference schemes. As a part of the study, it also develops discrete analogs of several notions of differential geometry, for example, Lagrangian and Hamiltonian formalisms and conservation laws.

The introductory part of the book surveys results concerning the application of Lie symmetry methods to differential equations. It then goes on to the sketching of examples of differential equations, namely the ordinary differential equation $u''=u^{-3}$ and the one-dimensional linear heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2u}{\partial t^2},$ what is to come in the rest of the book. It is explained that the symmetries of the former equation cannot be retained if it is discretised on a uniform mesh, and that the symmetries of the latter cannot be retained on an orthogonal mesh. This provides the motivation for the rest of the book, where in particular one can find constructions of difference schemes for which all the symmetries for the equations mentioned are retained. The rest can be summarised as follows.

Chapters 1 and 2 introduce discrete analogs of continuous differential geometric objects such as “discrete” vector fields. Chapter 3 presents group invariant difference schemes for most important ordinary partial differential equations, and introduces the notion of exact (i.e., approximating arbitrarily well) invariant difference scheme. Chapter 4 presents group-invariant difference schemes for partial differential equations such as the heat, Korteweg-de Vries, Schr?dinger. Chapter 5 discusses discretising delay differential equations and those coming from the Toda lattice model.

Chapter 6 (respectively 7) discusses Lagrangian (respectively Hamiltonian) formalisms for difference equations in the invariant setting, and Chapter 8 is devoted to the above mentioned exact schemes. The book is written in a concrete hands-on style, and can be recommended to specialists in difference schemes for differential equations and in difference equations. One shortcoming of the text is the absence of any discussion of numerical implementations. One might also find different parts of the text a bit disconnected from each other.

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