

## Approximations to Pi by the Kerala School of Mathematics

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Estimating the value of  $\pi$  (the ratio between the circumference and diameter of a circle) has been the favourite pastime of mathematicians from antiquity to the modern age. Most of the mathematicians in the ancient days used the procedure of inscribing and circumscribing the circle by polygons of a large number of sides and then approximating the circumference between the perimeters of the two polygons. This essentially requires one to find the side of the polygon as a function of the radius and consequent development of trigonometry. Although, this was the idea but the mathematicians didn't do it explicitly in terms of such trigonometric functions.

While the approximation for  $\pi = \frac{22}{7}$  was available from a long time, but many better results were also known to mathematicians of the ancient days, Archimedes (287-212 BC), perhaps one of the three greatest men of mathematics used a polygon of 91 sides to get  $\frac{10}{71} < \pi < \frac{10}{70}$ , which is an astonishing feat for that time. We may also mention in passing that the method used by Archimedes is termed as a precursor of integral calculus to be developed by Newton, Leibnitz and others about 1800 years later. Indian mathematicians Aryabhata in his Aryabhata (500 AD) gives approximately  $\pi = 3.1416$  by calculating the circumference of a circle of diameter 20000 to be 62832 while Bhaskara (~1114-1185) gives the circumference of a circle of radius 1250 to be 3927 getting approximately  $\pi = 3.14155$ .

Aryabhata

The ancient Kerala school of mathematics was remarkable in the sense that many discoveries of calculus and other related areas of mathematics can be found in the texts used and written by that school. One of the texts Yuktibhasha gives an exact infinite series expansion of  $\pi$ , not just another approximate value. This is indeed a giant conceptual leap and it doesn't appear anywhere else in the world upto 1600s, about 200 years after the Kerala school.

The series is now attributed to Gregory (1638-1675) and is,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

In fact, in addition to this result the text also gives an algebraic recursive method for finding  $\pi$ . In modern notation this corresponds to using the recursive relation

$$x_{n+1} = x_n^{-1} (\sqrt{1+x_n^2} - 1)$$
 with  $x_0 = 1$  and we obtain the limit,

$$\pi = 4 \lim_{n \rightarrow \infty} 2^n x_n$$

This method however requires one to evaluate the square root which is not an easy process, while the series expansion given in the text has no square roots.

References:

1. T. Padmanabham, Dawn of Science, 20. Calculus is Developed in Kerala, Resonance, Vol. 17, NO. 2, 106-115 pp.

**[Editor: Published on the occasion of  $\pi$  Day 2012.]**

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