Assam Academy of Mathematics Olympiad 2017 Questions

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The Assam Academy of Mathematics (AAM) organizes a mathematics Olympiad for school students every year. This year the Olympiad was held on 10th September, 2017 in three categories. We post below the questions for Category III (for students from classes 9, 10 and 11).

1. Prove that $latex A_n=5^n+2.3^{n-1}+1$ is a multiple of $latex 8$ for every positive integer $latex n$.
2. Prove that the diagonals of a quadrilateral are perpendicular if and only if the sum of the squares of one pair of opposite sides equal that of the other.
3. Prove that for every integer $latex n\geq 2$, $latex (1.2.3\ldots n)^2>n^n$.
4. Prove that $latex 11^{n+2}+12^{2n+1}$ is divisible by $latex 133$ for every natural number $latex n$.
5. Suppose $latex k,l,m$ are natural numbers. Prove that $latex 2^{k+l}+2^{k+m}+2^{l+m}\leq 2^{k+l+m+1}+1$.
6. Let $latex f:N\rightarrow M$ be a function such that (a) $latex f(m)<f(n)$ whenever $latex m<n$, (b) $latex f(2n)=f(n)+n$ for all $latex n\in N$ and (c) $latex n$ is a prime number whenever $latex f(n)$ is a prime number. Find $latex f(2001)$.
7. $latex S$ is a set of positive integers. None of the elements of $latex S$ is divisible by $latex n$. Prove that there exists a subset of $latex S$ such that the sum of its elements is divisible by $latex n$.
8. Consider a row of $latex n$ seats. A child sits on each. Each child may move at most by one seat. Find the number of ways that they can rearrange.
9. Let $latex P(x)$ be a polynomial over $latex \mathbb{Z}$. If $latex P(a)=P(b)=P(c)=-1$ with integers $latex a,b,c$, then prove that $latex P(x)$ has no integral zeroes.
10. Solve the equation $latex x^2-|3x+2|+x\geq 0$.
11. Show that the number $latex 10\cdots 01$ with $latex 1961$ zeroes is composite.
12. In the polynomial $latex x^3+px^2+qx+r$, one zero is the sum of the other two zeroes. Find the relation between $latex p,q$ and $latex r$.Questions 2 and 3 are of 8 marks each, question 4 is of 9 marks, questions 10, 11 and 12 are of 5 marks each, and the rest of the questions are of 10 marks each.

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