Experiences at a PhD Interview at TIFR-CAM

by Neeraj Singh Bhauryal - Friday, July 25, 2014

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This is the record of my interview held at TIFR Bangalore on 20th March'14 for Integrated program in Mathematics. So this was my second time at TIFR Bangalore ,last year I could not get through! This time I prepared much harder, mainly I did Real Analysis and I had more experience this time since in this one year I went through many exams and had lots of idea about what kind of questions can be asked. So Interview was at 9 in the morning and I was the first to appear that day.

I went inside the room and there were 5 Professors sitting inside, panel consists of Prof K Sandeep, Prof Venky Krishnan, Prof Praveen, Prof AS Vasudeva Murthy(don't remember the other one). I'll use P for Professor and I for myself.

I-Good Morning to all of you!

P-Good morning Neeraj, Welcome have a seat

P-So Neeraj you completed your BSc last year ,since what have you been doing?

I- Sir I joined Int-PhD program at IISER Mohali last year!

P-So why do you want to join us?

I- Sir I'm particularly interested in doing Analysis and I think TIFR-CAM is a good option for me

P- Okay so what courses have you done in 2nd sem?

I- Measure Theory, Complex Analysis, Combinatorial Group Theory, Discrete Mathematics, Groups and Fields, but I'm not much comfortable with my sem courses!

P-Let me ask some Measure Theory first, can you give an example of Lebesgue integral function which is not Riemann Integrable?

I- Yes, consider $$ f(x) =left{begin{array}{ll}x & mbox{if } x in mathbb{Q} \-x & mbox{if } x in mathbb{Q}^cend{array}right.$$

P-Correct, now can you give an example of function which is Reimann Integrable but not Lebesgue integrable?

I- Consider the integral $ int^infty_0 frac{sin x}{x},dx$, this integral is finite but $ int^infty_0 |frac{sin x}{x}|,dx$ is infinite.

P-Ok, find the Lebesgue integral of $$f(x)=left{begin{array}{ll}exp{x} & mbox{if } xin mathbb{Q}cap[0,1] \exp{-x} & mbox{if } x in mathbb{Q}^ccap[0,1]end{array}right.$$

I-(I had no idea about this, I thought I should write something on board and I knew what I'm writing is wrong!) Is it $ e^xmu(mathbb{Q}cap[0,1])+e^{-x}mu(mathbb{Q}^ccap [0,1])$

P- That's not correct you're using the formula for simple function! Let's leave it, Shall we start with Complex Analysis?

I-Sir i'm not much comfortable with that!

P-Okay give an example of sequence of continuous functions whose limit is not continuous.

I- Consider $$f_n(x) =left{begin{array}{lll}-1 & mbox{if } x < frac{-1}{n} \0 & mbox{if } frac{-1}{n} le x lefrac{1}{n} \1 & mbox{if} x>frac{1}{n}end{array}right.$$

This is continuous at $ x=0$ but limit function $ f(x)=sgn(x)$ not continuous at $ x=0$

P- I mean give an example of sequence of functions which are continuous on their domain but limit function is not continuous.

I- Then $ f_n(x)=x^n$ on $ [0,1]$ will work with limit function $$ f(x)=left{begin{array}{ll}0 & mbox{if } x in [0,1) \1 &mbox{if } x =1end{array}right.$$

P-Is it uniformly continuous on $ [0,1]$

I-No sir, since $ lim_{nto infty} sup_{xin [0,1]} |f_n(x)-f(x)|=1nrightarrow 0 $ as $ nto 0$

P-Okay, Is it uniform in interval $ [0,b]$ for some $$0<b<1$$

I- Yes, since $ lim_{nto infty} sup_{xin [0,b]} |f_n(x)-f(x)|=b^nto 0 $ as $ nto 0$

P-Yes, now consider a continuous function $ f(x):mathbb{R} to mathbb{R}$ satisfying $ |f(x)-f(y)|ge |x-y|$ and is bounded. Is it onto?

I-(I did this problem day before the interview but there $ f(x)$ was not bounded , and in excitement I didn't noticed this change and starting proving that $ f(x)$ is onto! As a result I got stuck in between and then I realized my mistake) Sir it can't be onto since it is bounded!

P-Can you give an example of such function.

I- $ f(x)= arctan x$ (I thought he's asking for bounded function which is not onto)

P-But does it satisfy the inequality property?

I-No because $ |f(x)-f(y)|ge |x-y| implies f'ge1 forall x in mathbb{R}$ which is not true here since $ f'(x)= frac{1}{1+x^2}le 1 forall x in mathbb{R}$

P- If a continuous function is mapping open intervals to open intervals will it map open sets to open sets(He asked me this because I used this somewhere while I was proving the result)

I-(I thought for a while)

P-(hints) every open set can be written as countable union of disjoint open intervals!

I- oh yes, then for any open set $ A=cup U_i$ for disjoint intervals $ U_i$, $ f(A)=cup f(U_i)$ and this union will be open!

P-Yeah that's true, now consider the sequence of functions $$ f_n(x)= left{begin{array}{ll}sin nx & mbox{if } 0le x le frac{pi}{2n} \0 & mbox{if } otherwiseend{array}right.$$

Is it pointwise convergent?

I- Yes, if we let $ nto infty$ then $ [0,frac{pi}{2n}]$ becomes $ {0}$ and $ f_n to 0$

P- Make it more precise.

I-(I was little confused exactly how to show it formally)

P-Fix $ x$ and then let $ n to infty$

I- Okay, so if we fix $ x=c$ then c comes out of interval $ [0,frac{pi}{2n}]$ for large n and so $ f_n(c) to 0$ pointwise.

P-Is the sequence uniformly converging to 0?

I-No sir , since $ lim_{nto infty} sup_{x in [0,frac{pi}{2n}]} |sin x| = lim 1 =1 nrightarrow 0$

P- Okay Neeraj we're done you may leave now!

I-Thank You very much.

[Neeraj is now an Integrated PhD student at the Centre for Applicable Mathematics, TIFR, Bangalore.]

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