Few Problems - 3

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1. Let $latex ABC$ be an acute angled triangle such that $latex \angle BAC = 45 ^o$. Let $latex D$ be a point on $latex AB$ such that $latex CD \perp AB$. Let $latex P$ be an internal point of the segment $latex CD$. Prove that $latex AP\perp BC$ if and only if $latex |AP| = |BC|$.
2. Show that there are infinitely many positive integers $latex n$ such that $latex n|3^{n}-1$.
3. Show that $latex n^5+n^4+1$ is always composite for all natural numbers $latex n > 1$. (Hint : Factorise $latex x^5+x^4+1$)
4. Show that the number of natural numbers divisible by $latex k\in \mathbb{N}$ less than or equal to $latex n\in\mathbb{N}$ is $latex \lfloor n/k \rfloor$.
5. For real numbers $latex a,b,c>0$ prove that
$latex 3(a^2+b^2+c^2)\geq (a+b+c)^2\geq 3(ab+bc+ca)$.

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