

## Few problems - 4

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1. Find all integers  $a, b, c$  satisfying  $1 < a < b < c$  such that  $(a-1)(b-1)(c-1)$  is a divisor of  $abc-1$ .
2. Find the number of ordered pairs  $(x, y)$  of positive integers which satisfy  $xy=27027$ .
3. The integer  $N$  consists of 2017 consecutive nines. A computer calculates  $N^3 = (999 \dots 999)^3$ . How many nines does the number  $N^3$  contain in total ?
4. Let  $ABC$  be an acute angled triangle such that  $\angle BAC=45^\circ$ . Let  $D$  be a point on  $AB$  such that  $CD \perp AB$ . Let  $P$  be an internal point of the segment  $CD$ . Prove that  $AP \perp BC$  if and only if  $|AP|=|BC|$ .
5. Let  $f(x)=x^n+5x^{n-1}+3$  where  $n > 1$  is an integer. Prove that  $f(x)$  cannot be expressed as the product of two non-constant polynomials with integer coefficients.

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