

## Few Problems - 1

by Gonit Sora - Tuesday, August 30, 2016

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1. Given a straight line  $l$  and points  $A$  and  $B$  on the same side of it find the shortest path from  $A$  to  $B$  which touches  $l$ .
2. Given a function  $f: \mathbb{Q} \rightarrow \mathbb{Q}$  such that  $f(x+y) = f(x) + f(y)$   
Show that  $f(x) = xf(1)$  for all  $x \in \mathbb{Q}$ . Thus, infer that for such a function knowing its value at one non-zero point is sufficient for finding its values at any point.
3. Find all integer values for  $x$ ,  $y$  and  $z$  satisfying the equation  $x^2 + y^2 + z^2 = 2xyz$ .
4. Given a triangle  $ABC$ , let  $AP$ ,  $BQ$  and  $CR$  be its altitudes. Show that  $AP$ ,  $BQ$  and  $CR$  are concurrent i.e. they meet a single point.
5. For a given prime number  $p$  find the number of quadruples  $(a, b, c, d)$  such that  $ad - bc \not\equiv 0 \pmod{p}$ .
6. There are six cities. Any two of them are connected by either a road or a railway track but not both. Show that there are at least three cities among these which are connected by a common mode of transport.

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