How to Fold It: The Mathematics of Linkages, Origami and Polyhedra

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http://gonitsora.com/fold-mathematics-linkages-origami-polyhedra/

Joseph O'Rourke

Cambridge University Press, 2011, xii+177 pp.

This book presents in three parts some geometric and physical aspects that have, for a long time, fascinated many people attracted by elegant structures provided by linkages, paper folding and polyhedral models.

Linkages used to be mainly associated with mechanical devices. They still do, of course — think of robots and simulation of human movements by robots. But now there is another aspect of linkages that some of us will learn for the first time from this book. It comes from one of the great biological revolutions of the past decades — the discovery and synthesis of complex protein molecules. Though the complexity of the molecular linkages of the real biological world is beyond a non-specialist's visualisation, the author shows us that some beautiful mathematical structure can be found in a very special type of linkages called "unit 90°-chains". We are pleasantly surprised that it is also related to a cute and stunning pop-up spinner constructed by a Japanese student in 1988.

There is a short chapter on origami, the visually captivating art and science of producing sculpture using just a sheet of paper. But it is not so much about making such paper sculptures as about the lesser known underlying mathematics, which is as elegant and beautiful as the physically and visually tangible end products themselves. The mathematics required is no more than basic 3-dimensional linear algebra, and the author offers at least outlines of mathematical proofs that will satisfy the mathematically minded reader. In the part on origami, we learn that there are some connections between paper folding and the so-called "one cuts" and dissections of polyhedra.

The third and final part on polyhedra deals with something that is perhaps not so well known to the nonspecialist. Ever since the discovery of the five Platonic solids of antiquity, mathematicians, sculptors and artists have been tantalised by solids with astonishing and intricate patterns of symmetry. Just as the surface of a cube can be "flattened" into a 2-dimensional plane without overlapping, these solids with some kind of symmetry can be similarly flattened into a connected non-overlapping "net" obtained by cutting along edges of the solid. This net can then be folded back and glued along appropriate edges to produce the original solid. However, for any convex polyhedron, this flattening procedure can be achieved provided the cutting is not restricted to the edges of the polyhedron. It requires what is known as an "Alexandrov gluing" — a deep and beautiful result discovered in 1942 by A D Alexandrov. We learn that while his proof was non-constructive, a new proof was recently found that can lead to an algorithm for the procedure. Unfortunately, this algorithm depends on solving a particular non-linear differential

equation, which can only be done by computational methods, that is, by using the computer.

This book can be readily read by someone with the most basic mathematical training that is normally provided in the first year at the university. If you are fascinated by 3-dimensional geometrical structures, you will enjoy learning the mathematical ideas behind them. There are exercises for practice and harder exercises for testing your understanding of the key ideas. Answers for these exercises are provided. Perhaps the more serious readerwould welcome more routine exercises to reinforce the concepts. This book incorporates some results found by the author and his students and poses some open problems in the field. The author shows us that there are areas in mathematics that offer opportunities for one to start doing research even with the most basic mathematical training. All that is needed is a curiosity into shapes and patterns and perhaps some geometric intuition. To help you get straight into the works, there are references to the principal textbooks and relevant research papers. The book has a website that will be useful to the beginner and interesting to the casual reader.

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