Indian National Mathematical Olympiad 2012

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The Indian National Mathematical Olympiad 2012 was held on 5th February, 2012 at various centres all over the country. The North East had three centres at Guwahati, Shillong and Agartala. The questions are given below:

1. Let $$ABCD$$ be a quadrilateral inscribed in a circle. Suppose $$AB=sqrt{2+sqrt{2}}$$ and $$AB$$ subtends $$135$$ degrees at center of circle . Find the maximum possible area of $$ABCD$$.

2. Let $$p_1<p_2<p_3<p_4$$ and $$q_1<q_2<q_3<q_4$$ be two sets of prime numbers, such that $$p_4$$ - $$p_1 = 8$$ and $$q_4$$ - $$q_1= 8$$. Suppose $$p_1 > 5$$ and $$q_1>5$$. Prove that $$30$$ divides $$p_1$$ - $$q_1$$.

3. Define a sequence $$langle f_n (x)rangle_{ninmathbb N_0}$$ of functions as
$$f_0(x)=1, f_1(x)=x, (f_n(x))^2-1=f_{n-1}(x)f_{n+1}(x), text{for} ngeq 1.$$
Prove that each $$f_n(x)$$ is a polynomial with integer coefficients.

4. Let $$ABC$$ be a triangle. An interior point $$P$$ of $$ABC$$ is said to be good if we can find exactly $$27$$ rays emanating from $$P$$ intersecting the sides of the triangle $$ABC$$ such that the triangle is divided by these rays into $$27$$ smaller triangles of equal area. Determine the number of good points for a given triangle $$ABC$$.

5. Let $$ABC$$ be an acute angled triangle. Let $$D,E,F$$ be points on $$BC, CA, AB$$ such that $$AD$$ is the median, $$BE$$ is the internal bisector and $$CF$$ is the altitude. Suppose that $$angle FDE=angle C, angle DEF=angle A$$ and $$angle EFD=angle B.$$ Show that $$ABC$$ is equilateral.

6. Let $$f : mathbb{Z} to mathbb{Z}$$ be a function satisfying $$f(0) ne 0$$, $$f(1) = 0$$ and

$$(i) f(xy) + f(x)f(y) = f(x) + f(y)$$

$$(ii)left(f(x-y)$$ - $$f(0)right) f(x)f(y) = 0 $$

for all $$x,y in mathbb{Z}$$, simultaneously.

$$(a)$$ Find the set of all possible values of the function $$f$$.

$$(b)$$ If $$f(10) ne 0$$ and $$f(2) = 0$$, find the set of all integers $$n$$ such that $$f(n) ne 0$$.

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