

Indian National Mathematical Olympiad 2018 Question Paper

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The Indian National Mathematical Olympiad (INMO) is a high school mathematical competition held annually in India since 1989. It is the second tier in the Indian team selection procedure for the International Mathematical Olympiad and is conducted by the Homi Bhabha Centre for Science Education (HBCSE) under the aegis of the National Board of Higher Mathematics (NBHM). The exam has six questions which have to be solved in four hours. The 33rd INMO was held on 21st January.

- Let $\triangle ABC$ be a non-equilateral triangle with integer sides. Let D and E be the respectively the midpoints of BC and CA ; let G be the centroid of triangle ABC . Suppose D, C, E, G are concyclic. Find the least possible perimeter of triangle ABC .
- For any natural number n consider a $1 \times n$ rectangular board made up of n unit squares. This is covered by three types of tiles: 1×1 red tile, 1×1 green tile and 1×2 blue domino. Let t_n denote the number of ways of covering $1 \times n$ rectangular board by these three types of tiles. Prove that t_n divides t_{2n+1} .
- Let Γ_1 and Γ_2 be two circles with respective centres O_1 and O_2 intersecting in two distinct points A and B such that $\angle O_1AO_2$ is an obtuse angle. Let the circumcircle of triangle O_1AO_2 intersect Γ_1 and Γ_2 respectively in points C and D . Let the line CB intersect Γ_2 in E ; let the line DB intersect Γ_1 in F . Prove that the points C, D, E, F are concyclic.
- Find all polynomials with real coefficients $P(x)$ such that $P(x^2+x+1)$ divides $P(x^3-1)$.
- There are $n \geq 3$ girls in a class sitting around a circular table, each having some apples with her. Every time the teacher notices a girl having more apples than both of her neighbors combined, the teacher takes away one apple from that girl and gives one apple each to her neighbors. Prove that this process stops after a finite number of steps. (Assume that the

teacher has an abundant supply of apples.)

6. Let \mathbb{N} denote the set of all natural numbers and let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a function such that: $f(mn) = f(m)f(n)$ for all m, n in \mathbb{N} ; and $m + n$ divides $f(m) + f(n)$ for all m, n in \mathbb{N} . Prove that there exists an odd natural number k such that $f(n) = n^k$ for all n in \mathbb{N} .

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