Indian National Mathematical Olympiad (INMO) 2014

by Gonit Sora - Monday, February 03, 2014

https://gonitsora.com/indian-national-mathematical-olympiad-inmo-2014/

The Indian National Mathematical Olympiad (INMO) was held on 2nd February 2014 throughout the country. The test was a four hour duration test, open for high school students who have already qualified the Regional Mathematical Olympiad (RMO). The questions of RMO 2013 can be found here. The questions asked in INMO 2014 are as follows.

1. In a triangle ABC, let D be the point on the segment BC such that AB + BD = AC + CD. Suppose that the points B, C and the centroids of triangles ABD and ACD lie on a circle. Prove that AB = AC.
2. Let n be a natural number. Prove that $latex \lfloor \frac{n}{1}\rfloor+\lfloor \frac{n}{2}\rfloor+\cdots+\lfloor \frac{n}{n}\rfloor +\lfloor \sqrt{n}\rfloor$ is even.
3. Let a, b be natural numbers with ab > 2. Suppose that the sum of their greatest common divisor and least common multiple is divisble by a + b. Prove that the quotient is at most $latex \frac{a+b}{4}$. When is this quotient exactly equal to $latex \frac{a+b}{4}$?
4. Written on a blackboard is the polynomial $latex x^2 + x + 2014$. Calvin and hobbes take turns alternatively(starting with Calvin) in the following game. During his turns alternatively(starting with Calvin) in the following game. During his turn, Calvin should either increase or decrese the coeffecient of $latex x$ by 1. And during this turn, Hobbes should either increase or decrease the constant coefficient by 1. Calvin wins if at any point of time the polynomial on the blackboard at that instant has integer roots. Prove that Calvin has a winning stratergy.
5. In a acute-angled triangle ABC, a point D lies on the segment BC. Let $latex O_1, O_2$ denote the circumcentres of triangles ABD and ACD respectively. Prove that the line joining the circumcentre of triangle ABC and the orthocentre of triangle $latex O_1O_2D$ is parallel to BC.
6. Let n be a natural number.And let X = {1, 2, ..., n} and define A?B to be the set of all those elements of X which belong to exactly one of A and B.Show that |F| ? 2n?1 where F is a collection of subsets of X such that for any two distinct elements of A, B of F,we have |A?B| ? 2.Also find all such collections F for which the maximum is attained.
