A mathematical marvel: Manjul Bhargava

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Dr. Anupam Saikia ***profiles prodigious mathematical talent of Indian origin, Manjul Bhargava, who was named a recipient of this year's Fields Medal, the highest honour in mathematics. This article was published in the Horizon supplement of The Assam Tribune on 10th October 2014 and has been reproduced here verbatim with the author's permission.***

Many of us are perhaps not aware of the fact the Nobel Prize is not awarded in the area of mathematics. Some people attribute it to the acute rivalry, a contemporary mathematician of great repute. A Canadian mathematician John Charles Fields took the major initiative of instituting an award for recognizing significant mathematical contributions. This award later came to be known as the Fields Medal. It was first awarded in 1936. The medal is awarded every four year during the International Congress of Mathematicians (ICM), a quadrennial event attended by more than three thousand mathematicians from all over the world. The Fields Medal prize money of 15,000 Canadian dollars is not comparable to the Nobel prize money of over one million US dollar. However, Fields Medal is regarded just as prestigious, if not more, as the Nobel prize in the community of mathematicians. The medal is awarded to mathematicians under the age of forty, and it is awarded once in four years. There have been occasions when mathematicians of the highest caliber had to miss out on the medal due to the age restriction.

The recipients of Fields Medal awarded this year during the ICM held in Korea included Manjul Bhargava, who is a mathematician of Indian origin. It often happens that in India we celebrate the success of persons of Indian origin even when they barely retain any connection with India. However, Indians can truly rejoice in Manjul Bhargava's achievements. Through his parents lived in Canada, Manjul used to visit India frequently from his early years and was greatly influenced by his grandfather in Rajasthan. He mastered Sanskrit at a tender age and also developed a keen interest in the Indian classical music. He became an expert player of the tabla and sitar and later he took Tabla lessons from the tabla maestro Zakir Husain. Manjul has also been closely associated with the community of mathematicians in India for more than a decade. he has held the position of Adjunct Professor at the Tata Institute of Fundamental Research, Bombay as well as at IIT-Bombay and University of Hyderabad. He has done collaborative research work with several Indian mathematicians like Eknath Ghate of TIFR therefore it is no surprise that the mathematicians in India have been overcome with joy ever since Manjul became the first person of Indian origin to win the Fields Medal.

Image Source : Infosys Science Foundation

Manjul enjoyed mathematics from a very young age. He recounts a childhood incident that triggered his imagination. Once he grew curious about the number of oranges required to construct a pyramid of a given height. His parents encouraged him to work it out by himself, and so he started playing with the oranges to get the right number and subsequently gave the precise mathematical argument for his answer. Manjul's mother Meera Bhargava taught mathematics at a university in Canada, He used to miss his

school to attend her classes. His prodigious talent was recognized early. He completed his graduation from Harvard University and then went to Pinceton University to purse his PhD under the guidance of none other than Andrew Wiles. He did groundbreaking research as a PhD student and widely acknowledged for that. Later he became a visiting fellow at Princeton's Institute for Advanced Study, a place where Albert Einstein had worked for more than two decades. Bhargava became a professor of Pinceton University at the age of 28, which is truly an extraordinary feat. It was no surprise at all to the community of mathematicians when he was awarded the Fields Medal. One of the most significant piece of work done by Manjul Bhargava was regarding quadratic forms. A binary quadratic form is a polynomial of the form $$f(x, y)=ax^2+bxy+cy^2$$ where a, b, c are fixed integers, and where x, y are two variables taking integer values. One interesting question is which integers are represented by the quadratic form as x and y range over all integral values. Fermat showed that prime numbers like 7 or 11 which leave a remainder 3 when divided by 4 can never be represented in the form $$x^2+y^2$$ wheres any prime number leaving remainder 1 after division by 4 can be written as x^2+y^2 for some choice of x and y in integers. For example, $$13=3^2+2^2$$ or $$17=4^2+1^2$$. Manjul proved that a quadratic form in two or more number of variables will represent any positive integer if it represents the first 290 natural numbers. He also prove that if such a form represents the first 21 primes, then it will represent any other prime.

It was the Indian mathematician Brahmagupta who observed in AD 826 that the product of the two quadratic forms $$x^2+y^2$$ and $$u^2+v^2$$ is again a quadratic form $$w^2+z^2$$ where w=xu+yv. Gauss, one of the greatest mathematicians of all time, generalized this in 1801 to prove that two binary quadratic form, and that the composition behaves just like multiplication of two non-zero rational numbers. This result by Gauss is known as Gauss Composition Law. For more than 200 years mathematicians could not find a way of generalizing Gauss's composition laws until Manjul's accomplishment. Manjul showed in his PhD work that Gauss Composition Law can be quadratic forms of higher number of variables too.

On a personal note, I always take pride in the fact I was fortunate enough to interact with Manjul Bhargava on a few occasions that while working as a postdoctoral fellow at McGill University in Montreal. I was not sure whether I can write about Bhargava's work in a popular newspaper, but was compelled by my deep respect for Professor Jyotiprasad Medhi who had asked me to do so. For the uninitiated, Professor Medhi has a very high standing in the world of mathematics and statistical sciences for his seminal contributions. What Prof. Medhi went on to accomplish after completing his college education in Assam several decades ago can be highly inspiring for the younger generation. Being a scholar of Sanskrit himself, Prof. Medhi perhaps appreciates it all the more that Bhargava often traces some of his key ideas in his work back to ancient Sanskrit texts. It is widely expected that Manjul Bhargava will continue to scale several peaks in his mathematical journey.

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