Mathematical Quest 2: Finding out square roots

by Gonit Sora - Monday, July 08, 2013

https://gonitsora.com/mathematical-quest-2-finding-out-square-roots/

The editor of the English section of Gonit Sora, writes a monthly column devoted to mathematics in a teen magazine called Young NE. The editor of Young NE was kind enough to grant us permission to republish the column after it appears on print. Below is the column that appeared in Vol 1, Issue 2, June 2013 of Young NE.

Finding out the square root of a number is a tedious job for high school students. The method given in the textbooks involve an intricate system of division and multiplication. But sometimes, while doing problems in general science or something else, one might be required to find an approximate square root of a number. How does one then find the square root of a number upto a given degree of accuracy failry quickly without reverting back to the traditional method. In this issue I shall present such a method.

Step 1: By guesswork assume a number to be the square root of the given number, say n.

Step 2: Divide n, with the guessed square root, say s. Let the result be r.

Step 3: Find the number (r+s)/2. This will be the new square root, and replace this number by s in the previous steps.

Step 4: Continue this process until the required accuracy is not obtained. To obtain p digits of accuracy after the decimal point you shall have to repeat the above process p-1 times.

Let me illustrate the above procedure with an example, so that things become easier to visualize. In fact, the above procedure is quite easy when you practise with it for some time. You will notice that you get the results quite faster too compared to the traditional method.

Example: Let us find the square root of 10.

Step 1: Let me guess the square root to be 3.

Step 2: Now, 10/3=3.3333 (I shall truncate it to 4 digits after the decimal point).

Step 3: (3+3.33)/2=6.33/2=3.165

Step 4: We replace 3 with 3.165 in the above steps.

[The actual square root is something like 3.162277... You notice that we have already obtained the square root to 2 correct decimal places.]

Let us repeat the process one more time.

Step 1: This time we take 3.165 as the square root.

Step 2: 10/3.165=3.1596 (I truncate it to 4 digits after the decimal point).

Step 3: (3.165+3.1596)/2=6.3246/2=3.1623

Step 4: We can again repeat the process like before.

We see that doing this process twice we get an accuracy of 3 digits after the decimal point. This much accuracy shall suffice for all practical purposes, but if the reader wishes then he or she may continue the above process arbitary number of times.

For the more advanced readers who wish to understand why the above process works, you can check the above iterative scheme using a sequence of numbers where say the first number is a, and there is a given number b. Then the number after a will be (a+b/a)/2 and so on. It will be an interesting exercise for the more advanced readers to show that this sequence actually will converge to the square root of b.

State Mathematical Olympiads: In India, mathematical olympiads are organised for high school students in two stages: Regional (RMO) and National (INMO). To participate in the RMO, students of Assam need to qualify the state level olympiad organised by the Assam Academy of Mathematics (AAM) every year in September. Details about that exam can be found in their website http://www.aam.org.in . Interested readers willing to know more about these exams may write to the author or to Prof. Mangesh B. Rege at mb29rege@yahoo.com.

Some important olympiad related resources can be downloaded at http://www.aam.org.in, http://www.mathlinks.ro, http://gonitsora.com, etc.

_____

PDF generated from https://gonitsora.com/mathematical-quest-2-finding-out-square-roots/.