# ''Mathematics Lab''

**by Debashish Sharma - Wednesday, July 20, 2016**

http://gonitsora.com/mathematics-lab/

Sounds strange, right? Well, "Mathematics Lab" is something we are not well-acquainted with. We all had "Physics lab", "Chemistry Lab", "Biology Lab" in our schools/colleges but not "Mathematics Lab". Last month, I and my friend Ajoy Dutta got an opportunity to design a "Mathematics Lab", thanks to Dr. Kedar Nath Das and Dr. S. K. Tripathy, faculty members of Mathematics and ECE departments respectively of  National Institute of Technology Silchar. They gave us a golden chance to conduct laboratory classes in Mathematics as a part of DST sponsored INSPIRE camp organized by NIT Silchar from 5th - 9th July 2016. Designing the "Mathematics Lab" turned out to be a very useful task for both of us as we got to learn so many new things.  We designed most of the activities from the excellent book "Mathematical Circles (Russian Experience)" by Dmitri Fomin, Sergey Genkin and Illia Itenberg. We also made good use of Pigeonhole Principle to design surprising activities. Each class was of three and half hours duration. We conducted four classes for four different groups of students in four different days. The problems set is attached at the end of this article as a pdf file. Some of the activities are described here.

## Pigeonhole Principle

This principle always gets the best response from the audience. We introduced this principle through an interesting activity. We gave blank paper chits to each student. There were almost 40 students in each class. We asked them to write any integer of their choice on the chits and then fold the chits. We collected the chits in a box. We then announced our claim that whenever any six chits are picked up randomly from the box, there will always exist two chits such that the difference of the numbers written on them is divisible by 5. The students were surprised with this claim. So, we asked the students to pick up six chits one by one and write the number on the board. Every time they ended up with two numbers whose difference is divisible by 5.  The underlying mathematics here is the Pigeonhole Principle which was also explained after the activity.

> If $n+1$ pigeons are to be placed in $n$ pigeonholes then one of the holes will have more than one pigeon

When a number is divided by 5, all possible remainders are 0,1,2,3 and 4. We consider these 5 remainders as 5 pigeonholes. Each of the six randomly picked number will be placed in one of these 5 pigeonholes. So it's like placing 6 pigeons in 5 pigeonholes. Hence there will be one pigeonhole having more than one pigeon i.e. there will exist two numbers which leave the same remainder when divide by 5 and hence their difference will be divisible by 5.

## Game with odd & even concept

We designed this game from a problem given in the book Mathematical Circles. We wrote the numbers

from 1 to 15 on the board. We asked two students to act as two players. In each turn a player puts a plus (+) or minus (-) sign in between the numbers, according to his choice. When all the spaces between the numbers are filled with + or - signs, we compute the resulting sum. The first player wins if the sum is even and the second player wins if the sum is odd. Who will win?

 Explaining the rules of the game to the players (Photo : Debashish Dash, PhD scholar, NIT Silchar)

No matter how the + and - signs are inserted, the first player will always win. This is because the sum of even number of odd numbers is always even !

## Game involving invariant

This game also from the same book. We wrote ten 1's and ten 2's are written on the blackboard and asked two students to act as players. They take turns in erasing any two numbers written on the board i.e in each turn a player deletes any two numbers. If the two numbers erased are identical, they are replaced with a 2. If they are different, they are replaced by 1. The game ends when only one number is left on the board. The first player wins if a 1 is left and the second player wins if a 2 is left on the board. Who will win?

Here, the first player will always win. If two identical numbers are erased, then the number of 1s either remain same or decrease by 2. And if two different numbers are erased then the number of 1s remains same. Thus in each turn the number of 1s either remains same or decreases by 2. That is, the parity of the number of 1s remains invariant in each turn. Since we started with even number of 1s, so the number of 1s will always remain even. So there cannot remain a single 1 at the conclusion of the game. Hence 2 will remain on the board when the game stops. Thus the second player will always win !

## Composition table for dihedral group of order 8

This activity is from the book "Contemporary Abstract Algebra" by J.A. Gallian. Suppose we remove a square from a plane, move it in some way and then place it back into the space it originally occupied. The goal of this activity is to describe all possible ways in this can be done, i.e. we need to describe the relationships between the starting position of the square and its final position. We provided each student with a 10cm by 10 cm square card with four corners having four colours (same colours back to back on both sides). We fixed the initial position $R_0$ (no change in osition) as the following :

 $R_0$

The other motions are rotation by 90 degree anticlockwise ($R_{90}$), rotation by 180 degrees anticlockwise ($R_{180}$), rotation by 270 degrees anticlockwise ($R_{270}$), rotation of 180 degrees about the horizontal axis passing through the centre of the card (H), rotation of 180 degrees about the vertical axis passing through the centre of the card (V), rotation of 180 degrees about the man diagonal ($D_1$) and rotation of 180 degrees about the other diagonal ($D_2$). We claim that any motion, however complicated, is equivalent to one of these eight positions. These eight motions together with the operation of composition forms a mathematical system called the dihedral group of order 8. It is denoted by $D_4$. After an explanation on how to compose two motions to obtain a

single motion, we asked the students to complete the following composition table :

 Composition table for Dihedral group of order 8

Most of the students completed the table correctly. After this we asked the students to construct a $3\times 3$ magic square and to apply the idea of above motions to construct all possible $3\times 3$ magic squares.

 Ajoy Dutta (PhD Scholar, Department of Mathematics, NIT Silchar) explaining magic squares

**Some interesting properties of Mobius strip**

When it comes to "Maths practicals", nothing can be better than a Mobius strip. Materials required for the Mobius strip experiment are

1. Rectangular strips of paper (We used the strips of Length 36 cm, width 5 cm, obtained by cutting an ordinary legal size paper)
2. Pen
3. Cellotape
4. Scissor

We first asked each student to make a cylinder using one of the supplied paper strips. Next we demonstrated the construction of a Mobius strip and asked each of them to do the same.

 A Mobius strip

 Cutting along the central line

The fact that the Mobius strip is a one-sided surface came as a surprise for the students. We asked  them to cut the cylinder along a line on the centre of the surface parallel to the edges. As a result they got two separate cylinders. Then we asked them to cut the Mobius strip along the central line parallel to its edge. The outcome was a single elongated strip with two twists.

Next we gave them a coloured Mobius strip as shown in the adjoining picture. We asked them to cut the Mobius strip along a line one-third of the way to one edge. They were mesmerized at the result - they got two separate strips this time, of two different colours, wound around one another. One was a smaller Mobius strip and the other was an elongated strip with two twists.

 Rectangular strip for constructing the coloured Mobius strip

 Coloured Mobius strip

Cutting along one third of the width of the strip from the edge

Interesting... isn't it ?

The complete problem set can be found <u>here</u> as a pdf file.

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