

# Navier-Stokes equation

by Manjil Saikia - Wednesday, July 25, 2012

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This is an equation which describes the motion of an incompressible fluid and is given by

$$1. \frac{\partial u_i}{\partial t} + \sum_{j=1}^3 u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + f_i(x,t) \quad x \in \mathbb{R}^3, t \geq 0, 1 \leq i \leq 3$$

$$2. \operatorname{div} u = \sum_{j=1}^3 \frac{\partial u_j}{\partial x_j} = 0 \quad x \in \mathbb{R}^3, t \geq 0$$

with initial conditions  $u(x,0) = u^0(x) \quad x \in \mathbb{R}^3$ . These have to be solved for an unknown velocity vector  $u(x,t) = (u_i(x,t))_{1 \leq i \leq 3} \quad x \in \mathbb{R}^3$  and pressure  $p(x,t)$  defined for position  $x \in \mathbb{R}^3$  and time  $t \geq 0$ . Here  $u^0(x)$  is a given infinitely differentiable divergence-free field and  $f_i(x,t)$  are the components of a given external applied force (e.g. gravity),  $\mu$  is a positive coefficient (viscosity) and  $\Delta = \sum_{j=1}^3 \frac{\partial^2}{\partial x_j^2}$  is the Laplacian in the space variables. Equation (2) is the incompressibility condition.

The equation with  $\mu$  set to 0 is called Euler equation and was derived by Euler in the eighteenth century from Newton's laws, applied to an incompressible fluid moving under an external force, free of any internal force i.e. viscosity. The viscosity term was introduced by Navier in 1822. His results were correct but his derivation was faulty and was corrected by Stokes in around 1842 and the equation henceforth called Navier-Stokes equation became the central equation in fluid mechanics. Approximate solutions of the equation are used in many applications in Physics and Engineering. However till now no one has proved or disproved that a solution of Navier-Stokes equation must exist for any values of  $u^0(x)$  and  $f(x,t)$ . This is then the problem of existence of solution of Navier-Stokes equation and which has been chosen as a millennium problem.

[This is the last article in our series on [Millennium Problems](#).]

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