On Finding the Next Term

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Having given a few terms of a sequence, the question : What is the next term? is frequently asked in examinations. For example, the question may be to find the next term in the sequence

6, 12, 24, 48, ... ———(1)

The given four terms can be easily seen to form a GP of common ratio 2.

So the usual answer will be that the next term (i.e. 5th term) will be 96. Take another example, that of finding the next terms in the sequence.

1, 2, 3, 5, 8, ... ———(2)

Here the given terms are neither in AP nor in GP. But one may notice that the 3rd term 3 is the sum of the first 2 terms, the 4th term 5 is the sum of the 2nd and the 3rd terms. Similar is the case with the 5th term 8. Hence it may be quite naturally and logically be said that the 6th term can be similarly taken to be the sum of 4th and 5th terms. Thus the next terms in (2) will be 5+8=13, as it is the case with the well known Fibonacci sequence.

1, 1, 2, 3, 5, 8, 13, … ———(3)

Now instead of the expected answer 96 in the first example (1), if the examinee's answer is 90 especially without any further clarification, then this answer will be considered wrong ordinarily. However, the examinee's line of thinking might have been different. He or she may have guessed or found the simple expression:

$$8n+n^2.(n-3)$$ ———(4)

which does yield the four terms in (1) for n equal to 1, 2, 3 and 4 respectively. Then by letting n=5, we get 90 as the next term of (1) as answered by the examinee. In addition to some sort of intuitive or mathematical vision, simple methods of getting the polynomials like (4) are those of finite differences, method of intermediate coefficients and Lagrange's method (which is instant expect for simplification).

The Fibonacci sequence (3) is a particular case of sequence defined by the recurrence relation

$$t_n=F(t_{n-1},t_{n-2})$$ ———(5),

Where $n>2$ with t_1 and t_2 given separately. For Fibonacci sequence (3), we have simply

$$t_n=t_{n-1}+t_{n-2}; t_1=t_2=1$$ ———(6).

Now let us try to get a relation of the type

$$t_n=p.t_{n-1}+q.t_{n-2}$$ ———(7).

For the given sequence (1), where p and q are unknown coefficients to be determined.

Here we have $$t_1=6; t_2=12.$$ Taking n=3 and n=4 in (7) and substituting values, we get

24=12p+6q ———(8),

48=24p+12q ———(9).

One solution of these equations is p=1 and q=2, and so we have, by (7),

$$t_n=t_{n-1}+2.t_{n-2}; n>2$$ ———(10)

This relation along with $$t_1=6$$ and $$t_2=12$$ enables us to get the sequence

6, 12, 24, 48, 96, 192, ... ———(11)

which is interestingly, same as the guessed G.P. An explanation for this is that the general term of (1) as G.P. is

$$t_n=6.2^{n-1}$$ ———(12)

which can be written as

$$t_n=6.2^{n-2}+2.6.2^{n-3}=t_{n-1}+2.t_{n-2}; n>2$$ ———(13)

which is same as (10). So a relation of the type (7) does not yield a new sequence for (1).

Let us try to find a relation of the type

$$t_n=r.(t_{n-1})(t_{n-2})+s; n>2$$ ———(14)

For (1) where $$r$$ and $$s$$ are to be found to be possible. By putting $$n=3$$ and $$n=4$$ in (14) and using $$t_1=6$$ and $$t_2=12$$ as before, we now get

$$24=72r+s$$ ———(15)

$$48=288r+s$$ ———(16)

Solving these we have as $$r=\frac{1}{9}$$ and $$s=16,$$ and thus (14) becomes

$$t_n=\frac{1}{9}.(t_{n-1})(t_{n-2})+16; n>2$$ ———(17)

In this way we are led to a new sequence

6, 12, 24, 48, 144, ... ———(18)

Here we find the 5th term to be 144 which is different from 96 (obtained from G.P.) and from 90 as well, which was obtained from the polynomial (4).

In a recent quiz book (Rao p-25) appears the question of finding the next term in

1, 2, 3, 5, 16, 231, ... ———(19)

The given answer **53105** is stated to have been obtained by noting that here

$$t_n=(t_{n-1})^2-(t_{n-2})^2; n>2$$ ———(20)

which is just another example of (5). What we have tried to illustrate is that there may be no end to various rules of formation of sequence, and that for finding the next term in a given sequence, there are several possibilities.

In fact and interestingly, it will be now shown that ANY number can be the next term in the usually given set of terms. Suppose we want $$k$$ to be the next term in (1). For this, one method will be to fit or find a 4th degree polynomial

$$y=f(n)$$ ———(21)

which will give the values 6, 12, 24, 48 and $$k$$ respectively for n=1, 2, 3, 4 and 5. By the usual method of finite differences, we easily get $$t_1=6; \Delta t_1=6; \Delta^2t_1=6; \Delta^3t_1=6; \Delta^4t_1=k-90.$$ Since we are fitting a 4th degree polynomial, the 5th order and higher order finite differences are to be taken zero. Then by usual interpolation formula

$$t_n=(1+\Delta)^{n-1}t_1$$ ———(22)

We get $$t_n=8n+(n-3).n^2+\frac{(n-1).(n-2).(n-3).(n-4).(k-90)}{24}$$ as the required general term. For n=1, 2, 3 and 4 this polynomial clearly reduce to (4) and gives the first four terms of (1). But for n=5, we have

$$t_5=90+24.\frac{k-90}{24}=k$$

as required. In this way ANY NUMBER can be made to be the next term in (1).

Theoretically there are many possibilities. So some understanding is essentially either implied or needed in connection with the question:

What is the Next Term?

References:

1) R. C. Gupta. “What is the next term?”, Mathematics teacher (India), Vol 20 (1985), pp 11-13.

2) D Jagan Mohan Rao, The Master Book of Mathematics Quiz, Neelkamal Publications, Hyderabad, 2003.

Note:

The above article was published in the “Indian Journal of Mathematics Teaching” (Vol 31 Numbers 1 & 2 year - 2005) by association for improvement of Mathematics Teaching, Kolkata. Such is the need of the hour that he would to publish the same article in JM. As he rightly pointed out, the mode of several talent exams have a wrong idea about “find the next term” and such exams increase day by day. It is ultimately important for the student to know that Maths is a study of Facts and Facts on Figures but not the converse.

In a recently conducted talent exam, a problem was posed in the “choose the correct answer” column:

Find the next term: 12, 24, 38, 54, 72,.…

a) 90 b) 96 c) 92 d) 98

The fact is that all the options are correct as they could the next term of the given undefined sequence by the rule:

nth term is $$t_n=n.(n+9)+2+\frac{(n-1).(n-2).(n-3).(n-4).(n-5)}{120}(w-92)$$ where w stands for wishing option.

Beware: An undefined sequence taken its own form.

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