Poincare Conjecture

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[Editor's Note: **Jules Henri Poincaré** (29 April 1854 – 17 July 1912) was
a French mathematician, theoretical physicist, engineer, and a philosopher of science. He is often
described as a polymath, and in mathematics as *The Last Universalist*, since he excelled in all fields of the
discipline as it existed during his lifetime.

This is the second problem discussed under the Millennium Problem series.]

Since the days of Newton, the notion of continuity and limits entered Mathematics and these notions were
brought in to explain motion. As a result, the notion of continuous functions was formulated where
$$f(y)$$ goes near to $$f(x)$$ when $$y$$ goes near to $$x$$. For the concept of nearness, a distance
function $$d(x,y)$$ can be introduced. For example for two points $$x, y$$ in a rubber sheet we can take
$$d(x,y)$$ to be the actual distance of $$x$$ and $$y$$. However rubber sheets can be stretched altering
the actual distances but the notion of continuous functions should remain same. A topological space is
defined to be a set having certain subsets called open sets. The collection of open subsets satisfies certain
simple properties and is called a topology for the original set. A subset containing an open set containing
a point $$x$$, is called a neighbourhood of $$x$$ and the notion of continuity was captured as the
property that given a neighbourhood $$V$$ of $$f(x)$$, there is a neighbourhood $$U$$ of $$x$$ such
that if $$y$$ is in $$U$$ then $$f(y)$$ is in $$V$$. After topological spaces were established as the
proper framework to discuss continuity, Poincare went ahead to classify some useful topological spaces
that we encounter all the time. For this classification, the notion of a homeomorphism was introduced.
Two topological spaces $$X$$ and $$Y$$ are called homeomorphic if they are essentially carbon copies
of each other i.e. there is an one-to-one onto functions from one to the other which are inverses of each
other and take open sets to open sets.

Now to classify topological spaces, Poincare looked for certain standard topological spaces. One such
space is an open ball in $$R^n$$ of radius $$r>0$$ and center $$xinR^n:B_n(x,r)=\{yinR^n mid mid
midy–xmidmid<r\}$$. A topological space is then called a manifold if every $$xinX$$ has a
neighbourhood homeomorphic to an open ball in $$R^n$$ for some $$n$$. The value of $$n$$ is then
called the dimension of the manifold. Topological spaces can also satisfy another property called
compactness which means that if the space is covered up by a collection of open sets, it can be covered
up by a finite subcollection. Compact topological spaces display a lot of pleasant properties which give
them their importance.

Poincare wanted to classify compact manifolds. One such manifold is the n-dimensional sphere which is
the surface of the unit sphere in $$R^{n+1}$$ i.e. numbers $$(x_1,x_2,ldots,x_{n+1}$$ such that

$$\sum_{i=0}^{n}x_i=1$$. It is easy to prove that for $n\geq2$ the n-dimensional sphere S^n is such that any closed curve in that set can be squeezed to a point. This is called simple connectivity. i.e. for $n\geq2$, S^n is simply connected. Thus if X is homeomorphic to S^n for $n\geq2$, then it is simply connected. Poincare raised the question of the converse. For $n=2$, it is simple to prove that any simply connected compact 2-dimensional manifold is homeomorphic to S^2. Poincare then made the conjecture for a similar result for S^3. Actually his conjecture could be extended to all values of $n>3$. In 1960, American mathematician Stephen Smale proved the conjecture for all values of $n>4$. However his methods did not work for $n=3$ and 4. In 1981, another American, Michael Freedman proved the conjecture for $n=4$. For these results Smale and Freedman were awarded the Fields Medal the highest award for original work to mathematicians below the age of forty. However Poincare's original conjecture for $n=3$ remained unsolved and was named as a millennium problem in the year 2000. But just in three years, in 2003 Grigori Perelman of St Petersburg, Russia solved the problem. He was awarded the Fields Medal in 2006, and the millennium prize was awarded to him in 2010. He refused both of them.

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