

# Quantum Yang-Mills Theory

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[Editor: This is next part in our series of articles on the [Millennium Problems](#).]

In classical physics, there were two kinds of entities, material particles governed by Newtonian mechanics and fields governed by appropriate field equations eg Maxwell's equations for electromagnetic field. However just before and after the beginning of the twentieth century, experimental results indicated that fields (waves) can exhibit particle properties and particles can exhibit wave properties. For the explanation of the second situation, physicists introduced the notion of wave-functions which were fields and formed vectors in a Hilbert space eg  $L^2(\mathbb{R}^3)$ . The observable quantities were Hermitian operators in that space with their eigenvalues giving the observed values. To handle the first situation i.e. to understand the quantum aspects of fields, the fields themselves were treated as operators which acted on a Hilbert space of vectors which had particle-like excitations. The theory predicted that every particle must have an antiparticle of the same mass and opposite charge, and surely enough this was confirmed by the discovery of the positron. With this approach Maxwell's theory gave rise to a highly successful Quantum Field Theory called Quantum Electrodynamics (QED).

Maxwell's field equations had a feature called gauge invariance which was not noticed earlier but became important in its quantum-mechanical version Quantum Electrodynamics. The gauge transformations formed a group of symmetry of the system and in the case of QED, the gauge group is  $U(1)$  the multiplicative group of complex numbers of absolute value one. In 1954, physicists Yang and Mills proposed Yang-Mills theory which was a field theory with the gauge group being any simple compact Lie group (may be nonabelian). It was expected that this theory will be able to unify all the four fundamental forces of nature, the gravitational, the electromagnetic, the strong force and the weak force in a single unified quantum field theory. There were theoretical difficulties in these but overcoming them the weak force was brought under such a theory by the gauge group  $SU(2)$ , and later the strong force was similarly unified using the gauge group  $SU(3)$ . For the gravitational field more experimental results and theoretical developments will be needed. There is another peculiarity of such a theory which is not properly understood. The electromagnetic and the gravitational fields are carried by particles with zero rest mass and therefore can have a long range according to the theory. But the short and the weak forces are short range forces and therefore must have a positive rest mass. The mass is same as the energy whose eigenvalues are nonnegative. The theory works fine for any gauge group only when the mass is zero, or greater than a number  $\mu > 0$ . This number is called the mass gap. It is not clear to the physicists why this mass gap occurs but because of the immense experimental success, physicists are confident that all the difficulties will eventually be sorted out and a theory will be obtained which explains all the physical interactions occurring in this universe.

But the mathematicians are far from satisfied mainly because it is not known whether solutions exist for the Yang-Mills equations i.e. whether a Yang-Mills theory exists for any compact simple Lie group.

Physicists are after all computing and verifying some approximate solutions assuming that such solutions exist. Also the question of the mass gap is not clear. The situation is similar to that of Navier-Stokes equation. The whole of classical Physics also required a correct definition of a real number which was arrived at by the mathematicians only towards the end of the nineteenth century. Because of all these the following millennium problem has been proposed.

Yang Mills Existence and Mass Gap Problem : Prove that for any compact simple Lie group  $G$ , a non-trivial quantum Yang-Mills theory exists on  $\mathbb{R}^4$  (space-time) and predicts a mass gap  $\mu > 0$ .

If this theorem is proved then the whole of Modern Physics will start having a strong Mathematical foundation.

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