Regional Mathematical Olympiad – 2013

by Gonit Sora - Tuesday, December 03, 2013

https://gonitsora.com/regional-mathematical-olympiad-2013/

Time: 3 hours

December 01, 2013

Instructions:

· Calculators (in any form) and protractors are not allowed.

· Rulers and compasses are allowed.

· Answer all the questions.

· All questions carry equal marks. Maximum marks: 102.

· Answer to each question should start on a new page. Clearly indicate the question number.

1. Let $latex ABC$ be an acute-angled triangle. The circle $latex \Gamma$ with $latex BC$ as diameter intersects $latex AB$ and $latex AC$ again at $latex P$ and $latex Q,$ respectively. Determine $latex \angle BAC$ given that the orthocenter of triangle $latex APQ$ lies on $latex \Gamma.$

1. Let $latex f(x)=x^3+ax^2+bx+c$ and $latex g(x)=x^3+bx^2+cx+a$, where $latex a,b,c$ are integers with $latex c\neq 0.$ Suppose that the following conditions holds:

(a) $latex f(1)=0;$

(b) the roots of $latex g(x)=0$ are the square of the roots of $latex f(x)=0.$

Find the value of $latex a^{2013}+b^{2013}+c^{2013}.$

1. Find all primes $latex p$ and $latex q$ such that $latex p$ divides $latex q^2-4$ and $latex q$ divides $latex p^2-1.$

1. Find the number of 10-tuples $latex (a_1,a_2,\dots ,a_{10})$ of integers such that $latex |a|\leq 1$ and

$latex a_1^2+a_2^2+\ldots +a_{10}^2-a_1a_2-a_2a_3-a_3a_4-dots -a_9a_{10}-a_{10}a_1=2.$

1. Let $latex ABC$ be a triangle with $latex \angle A=90^{\circ}$ and $latex AB=AC.$ Let $latex D$ and $latex E$ be points on the segment $latex BC$ such that $latex BD:DE:EC=3:5:4.$ Prove that $latex \angle DAE=45^{\circ}.$

1. Suppose that $latex m$ and $latex n$ are integers such that both the quadratic equations $latex x^2+mx-n=0$ and $latex x^2-mx+n=0$ have integer roots. Prove that $latex n$ is divisible by 6.
