

Regional Mathematical Olympiad India 2017 Questions

by Manjil Saikia - Saturday, October 14, 2017

<http://gonitsora.com/regional-mathematical-olympiad-india-2017-questions/>

The [Regional Mathematical Olympiad](#) (RMO) is organized every year in India throughout the country as a preliminary screening for the [Indian National Mathematical Olympiad](#) (INMO). This year it was held on 8th October, 2017. The questions of this year's RMO can be found below.

(All questions carry equal marks, maximum possible score is 102.)

- Let $\angle AOB$ be a given angle less than 180° and let P be an interior point of the angular region determined by the angle $\angle AOB$. Show, with proof, how to construct using only ruler and compass, a line segment CD passing through P such that C lies on the ray AO and D lies on the ray OB , and $CP:PD=1:2$.
- Show that the equation $a^3+(a+1)^3+(a+2)^3+(a+3)^3+(a+4)^3+(a+5)^3+(a+6)^3=b^4+(b+1)^4$ has no solutions in integers for a, b .
- Let $P(x)=x^2+\frac{1}{2}x+b$ and $Q(x)=x^3+cx+d$ be two polynomials with real coefficients such that $P(x)Q(x)=Q(P(x))$ for all real x . Find all the real roots of $P(Q(x))=0$.
- Consider n^2 unit squares in the xy -plane centered at point (i, j) with integer coordinates, $1 \leq i \leq n, 1 \leq j \leq n$. It is required to colour each unit square in such a way that whenever $1 \leq i < j \leq n$ and $1 \leq k < l \leq n$, the three squares with centres at $(i, k), (j, k), (j, l)$ have distinct colours. What is the least possible number of colours needed?
- Let Ω be a circle with chord AB which is not a diameter. Let Γ_1 be a circle on one side of AB such that it is tangent to AB at C and internally tangent to Ω at D . Likewise, let Γ_2 be a circle on the other side of AB such that it is tangent to AB at E and internally tangent to Ω at F . Suppose the line DC intersects Ω at $X \neq D$ and the line FE intersects Ω at $Y \neq F$. Prove that XY is a diameter of Ω .
- Let x, y, z be real numbers, each greater than 1. Prove that $\frac{x+1}{y+1} + \frac{y+1}{z+1} + \frac{z+1}{x+1} \leq \frac{x-1}{y-1} + \frac{y-1}{z-1} + \frac{z-1}{x-1}$.

PDF generated from <http://gonitsora.com/regional-mathematical-olympiad-india-2017-questions/>.

This work is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.