Some Beautiful Mathematical Gems (Sequences)

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Mathematics is full of beautiful results, facts and theorems. One of the fun things to do in mathematics is to play with the numbers. Anyone enthusiastic enough can play with numbers and thereby discover some beautiful things hidden within it. It is always fun to play with numbers and many well known results in mathematics today is just the result of such fun investigation of numbers. Number theory is full of such beauties of numbers. In this article let us look at few such facts or results or whatever you want to call it.

Look and say sequence: In mathematics, we encounter sequences everywhere. We all know what a sequence is. Let alone the rigorous mathematical definition, a sequence is something that comprises of terms that follows each other in a somewhat ordered fashion. For example, $$1, 2, 3, 4, 5,ldots$$ is the sequence of Natural numbers. Again $$2, 3, 5, 7, 11,ldots$$ is the sequence of Prime numbers. We have some beautiful sequences in mathematics. Among them the most popular and fascinating sequence is the Fibonacci sequence, which can be written as $$1, 1, 2, 3, 5, 8, 13,ldots$$ etc. The next term in the Fibonacci sequence can be obtained by adding the previous two terms. For example the next term after $$13$$ is $$8+13=21$$. So one can easily predict the next term in the sequence. But as we know that mathematics can be weird sometimes. There are few sequences which defies common sense, that is they are not readily apparent to everyone as the sequence of prime numbers or the Fibonacci sequence. Let us take a look at this sequence called *the look and say sequence* (the meaning of the name will be apparent in a minute)

So here are first few terms of the sequence $$1,11,21,1211,111221,ldots$$. So the question is what about the next term? Looking at the sequence for the first time it doesn't exhibit any pattern but taking a closer look might reveal something interesting. Instead of trying to find the next term (and pulling your hair!) let's say the terms of the sequence, so the first term says "one", how many? One $$1$$. Hence the next term $$11$$. Now let's say the second term, it's "two $$1$$". So the third term is $$21$$ that is "two $$1$$s". So the terms of the sequence are what we got by saying the previous term, hence the name *look and say sequence.*

This sequence was introduced by John Conway and he had work on it to give further results. In particular, if $$L_{n}$$ denotes the number of digits in the $$n$$-th member of the sequence than the limit $$frac{L_{n+1}}{L_{n}}$$ exist and equals to $$lambda$$, where $$lambda=1.303577269034cdots$$ known as Conway's constant and is an algebraic number of degree $$71$$, that is $$lambda$$ is a root of the following polynomial!

$$x^{71}-x^{69}-2x^{68}-x^{67}+2x^{66}+2x^{65}+x^{64}-x^{63}-x^{62}-x^{61}-x^{60}-x^{59}+2x^{58}+5x^{57}$$
$$+3x^{56}-2x^{55}-10x^{54}-3x^{53}-2x^{52}+6x^{51}+6x^{50}+x^{49}+9x^{48}-3x^{47}-7x^{46}-8x^{45}-$$

$$8x^{44}+10x^{43}+6x^{42}+8x^{41}-5x^{40}-12x^{39}+7x^{38}-7x^{37}+7x^{36}+x^{35}-3x^{34}+10x^{33}+$$
$$x^{32}-6x^{31}-2x^{30}-10x^{29}-3x^{28}+2x^{27}+9x^{26}-3x^{25}+14x^{24}-8x^{23}-7x^{21}+9x^{20}+$$
$$3x^{19}-4x^{18}-10x^{17}-7x^{16}+12x^{15}+12x^{15}+7x^{14}+2x^{13}-12x^{12}-4x^{11}-2x^{10}$$
$$+5x^9+x^7-7x^6+7x^5-4x^4+12x^3-6x^2+3x-6$$.

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Lycherl Numbers: Before going to define what a Lycherl number is, first let's play with some numbers. Let us take a two digit number say 25, now what you have to do is swap the digits and add the resulting number with the previous one. So we get $25+52=77$. So what? Well 77 is a palindrome number, that is it looks same no matter how you swap the digits. Palindrome also exists in words like *radar, level* or more famous examples like *"A man, a plan, a canal, Panama!"* (the last one is a title of a tv show by NOVA!). Now in our first example, we got a palindrome number in just one step. We can also try with another number say 58. Then doing the iterations we get $58+85=143+341=484$. So we get a palindrome number after second iterations. All the two digits numbers becomes a palindrome number after a finite number of iterations. And also most of the three or higher digits numbers too become palindrome after finite iterations. But as anyone can expect, there are exceptions. The number 196 is a special number in this chain. No matter how many times you do the iterations, 196 never becomes a palindrome number. Though this fact hasn't been proven yet, but iterations upto billions of times haven't been able to produce one. So everyone is suspecting that 196 is not a palindrome number. Now the numbers which cannot be converted into a palindrome number are called Lycherl Numbers. The name Lycherl, coined by Wade Van

Landingham, is an anagram of his girlfriend's first name Cheryl. (Remember Cheryl's birthday problem?) So it is conjectured that 196 and other numbers that have not yet yielded a palindrome are Lychrel numbers, but no one has ever been able to prove that, so these are the 'candidate' Lycherl numbers. Few candidate Lycherl numbers are $196, 295, 394, 493, 592, 689, 691, 788, 790, 879, 887, 978, 986, 1495, 1497,$
$1585, 1587, 1675, 1677, 1765, 1767, 1855, 1857, 1945, 1947, 1997$. 196 is the smallest Lycherl candidate number. Do any base 10 Lycherl number exists? It is an open unsolved problem is mathematics. People have proved that there are Lycherl numbers in other bases (for example in base 2, 10110 is a Lycherl number!).

So we have seen that there are beautiful things that can emerge from playful investigations of simple numbers and with interesting properties. We will see more such beauties in future articles. All these things proved only just one thing, **Mathematics is beautiful!**
