Some Favourite Mathematical Constants

by Manjil Saikia - Tuesday, July 31, 2012

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Mathematical constant are really exciting and wonderful in the world of numbers. All numbers are not created equal; that certain constants appear at all and then echo throughout mathematics, in seemingly independent ways, is a source of fascination. Just as physical constants provide “boundary conditions” for the physical universe, mathematical constants somehow characterize the structure constants, the Archimedes’ constant $$pi$$ (pi) was once regarded as the king, and in modern times (after 1980), the Feigenbaum Universal constant $$delta=4.6692016091029dots$$ is regarded as the ‘Super King’ in this field on the basis of its increasing importance and tremendous uses in mathematical sciences, physics, chemistry, biosciences, economics, engineering etc.

The constants listed below are rather arbitrarily organized by topicwise. The concrete elaboration of how these constants are derived needs a long discussion, and hence detailed discussion is intentionally omitted. Interested readers are requested to contact the author for details of any constant. However, how the Feigenbaum Universal constant can be obtained is very briefly highlighted below with the help of a simple example.

Let $$f(x)=ax(1-x),$$ where a is a constant. The interval $$[0,1]$$ is mapped into itself by $$f$$ for each value of $$ain [0,4].$$ This family of functions, parameterized by a, is known as the family of **logistic maps**. What are the 1-cycles (i.e. fixed points) of f? Solving x=f(x) we obtain

x=0 (which attracts for a<1 and repels for a>1),

and $$x=frac{a-1}{a}$$ (which attracts for 1<a<3 and repels for a>3).

What are the 2-cycles of f? That is, what are the fixed points of the iterate $$f^{2}$$ which are not fixed points of f? Solving $$x=f^{2}(x), xneq f(x),$$ we obtain the 2-cycle.

$$x=frac{a+1pm (a^{2}-2a-3)^{frac{1}{2}}}{2a}$$ (which attracts for $$3<a<1+sqrt{6}$$ and repels for $$ a>1+sqrt{6}$$).

For $$a>1+sqrt{6}=3.4495dots$$ an attracting 4-cycle emerges. We can obtain the 4-cycle by numerically solving $$x=f^{4},$$ and $$xneq f^{2}.$$ It can be shown that 4-cycle attracts for 3.4495….<a<3.5441…., and repels for a>3.5441…..

For a>3.5441…., an attracting 8-cycle emerges. We can obtain the 8-cycle by numerically solving $$x=f^{8},$$ and $$xneq f^{4}.$$ It can be shown that 8-cycle attracts for 3.5441….<a<3.5644…., and repels for a>3.5644…..

For how long does the sequence of period doubling bifurcations continue? It's interesting that this behavior stops for short of 4. Setting $$a_{0}=1,a_{1}=3,a_{2}=1+sqrt{6}=3.4495dots , a_{3}=3.5441dots ,a_{4}=3.5644dots ,$$ and so forth denote the cascade of bifurcations, it can be proved that

$$a_{infty}=lim_{infty}a_{n}=3.5699dots <4.0$$

This point marks the separation between the "periodic regime" and the "chaotic regime" for this family of quadratic functions. The sequence $${a_{n}}$$ behaves in a universal manner such that the ratio $$frac{a_{n}-a_{n-1}}{a_{n+1-a_{n}}}$$ tends to a universal constant $$delta =4.6692016091029dots .$$

The elementary particle theorist, Mitchell J. Feigenbaum working in the University of Princeton, U.S.A, has explain in details the creation of this constant in his two marvelous papers.

(i) Quantitative Universality for a class of Nonlinear Transformations, J. of Statistical Physics, 19:1(1978), 25-52.

(ii) The Universal Metric Properties of Nonlinear Transformations, J. of Statistical Physics, 21:6(1979), 669-706.

Favourite Mathematical constants so far we know are listed bellow with their approximate numerical values:

	Well-known constants	Numerical Value
1	Zero	0
2	One	1
3	Imaginary unit	$$sqrt{-1}$$
4	Pythagoras' constant	$$sqrt{2}$$
5	Golden mean	$$phi=1.6180339887dots$$
6	Natural logarithmic base	C=2.7182818285….
7	Archimedes' constant	$$pi=3.14159265358979dots$$
8	Euler-Mascheroni constant	$$gamma=0.5772156649dots$$
9	Ape''ry's constant	$$zeta(3)=1.202056903dots$$
10	Catalan's constant	G=0.915965594….
11	Khintchine's constant	K=2.68545200….
12	Feigenbaum constant	$$delta=4.6692016091029dots$$
13	Madelung's constant	$$M_{2}=-1.6155426267dots$$
14	Chaiten's constant	Not available

Constants associate with Number Theory

15	Hardy-Littlewood constant	$$C_{twin}=0.6601618158dots$$
16	Hadamard-de la Valle’e Poussin constant	$$C_{1}=0.2614972128dots$$
17	Landau-Ramanujan constant	K=0.764223653….
18	Brun’s constant	B=1.90211605778….
19	Artin’s constant	$$C_{Artin}=0.3739558136dots$$
20	Linnik’s constant	Not available
21	Hafner-Sarnak-MeCurley constant	$$D_{infty}=0.3532363719dots$$
22	Gauss-Kuzmin-Wirsing constant	$$lambda=0.3036630029dots$$
23	Stolarsky-Harborth constant	$$Theta=1.58496dots$$
24	Porter’s constant	C=1.4670780794….
25	Glaisher-Kinkelin constant	A=1.28242713….
26	Franse’n-Robinson constant	2.8077702420….
27	Allodi-Grnstead constant	0.809394020534….
28	Niven’s constant constant	C=1.705211….
29	Backhouse’s constant	1.456074485826….
30	Mill’s constant	C=1.3064….
31	Stieltjes constant	$$gamma_{0}=0.5772156649dots$$
32	Liouville-Roth constant	0.0110001000….
33	Diophantine approximation constant	$$gamma_{1}=0.4472135955dots$$
34	Erdos reciprocal sum constant	3.0089
35	Abundant number density constant	0.2441<A<0.2909
36	Self-number density constant	$$lambda=0.252660259dots$$
37	Cameron’s sum-free set constant	0.21759<c<0.21862
38	Euler totient function asymptotic constant	A=1.9435964368….B=-0.0595536246….
39	Nielson-Ramanujan constant	Not available
40	Triple-free set constant	0.6135752692….
41	De-Bruijn-Newman constant	Not yet available
42	Freiman’s constant	Not yet available
43	Cahen’s constant	Not yet available

Constants associate with Analytic Inequalities

44	Shapiro’s cycle sum constant	0.4945668….
45	Carlson-Levin constant	$$Gammafrac{1/4}{pi ^{1/4}}$$
46	Londau-Kolmogorov constant	$$C(3,1)=(frac{243}{8})^{1/3}$$
47	Hilbert’s constant	Not available
48	Copson-de-Bruijn constant	C=1.1064957714….
49	Wirtinger-Sobolev isoperimetric constant	Not available
50	Whitney-Mikhlin extension constant	2.05003

Constants associate with the Approximation of Functions

51	Wilbraham Gibbs constant	G=1.851937052….
52	Lebesgue constant	C=0.9894312738….
53	Favard constant	Not available
54	Bernstein’s constant	$$beta=0.2801694990dots$$
55	The “one-ninth” constant	0.1076539192….
56	Laplace limit constant	$$lambda=0.6627434193dots$$

Constants associate with Enumerating Discrete structures

57	Abelian group enumeration constant	A=2.2948566…. , B=1.3297682….
58	R’enyi’s parking constant	0.7475979203….
59	Golomb Dickman constant	$$lambda=0.6243299885dots$$
60	Lengyel’s constant	$$lambda=1.0986858055dots$$
61	Otter’s tree enumeration constant	$$beta=0.5349485dots$$
62	Polya’s random walk constant	$$rho=0.3405373296dots$$
63	Self-avoiding-walk connective constant	2.6381585….
64	Feller’s coin tossing constant	$$alpha=1.087378025dots ,beta=1.236839845dots$$
65	Har square entropy constant	k=1.503048082….
66	Binary search tree constant	Precise numerical value not available
67	Digital search tree constant	c=0.3720486812….
68	Quardtree constant	C=4.3110704070….
69	Extreme value constant	$$mu_{6}=1.2672063606dots$$
70	Pattern-free word constant	$$varsigma=1.302dots$$
71	Takeuchi-Prellberg constant	c=2.239433104….
72	Random percolation constant	$$K_{B}=0.0355762113dots$$
73	Lenz-Ising constant	$$rho=0.218094dots$$
74	2D Monomer-dimer constant	1.338515152….
75	3D Dimer constant	$$lambda=0.209174dots$$
76	Lieb’s square ice constant	Not yet available

Constants associate with Functional Iteration

77	Gauss’s lemniscate constant	0.83462684167….
78	Grossman’s constant	Not available
79	Plouffe’s constant	0.4756260767….
80	Lehmer’s constant	0.5926327182….
81	Iterated exponential constant	-0.7666646959….
82	Continued fraction constant	0.76519769…., 0.7098034428….
83	Infinite product constant	2.0742250447….
84	Quadratic recurrence constant	C=1.502836801….
85	Conway’s constant	$$lambda=1.303577269dots$$

Constants associate with Complex Analysis

86	Bloch-Landau constant	B=0.4718617….L=0.5432588….
87	Masser-Gramain constant	C=0.6462454398….
88	John constant	4.810477….

Constants associate with Geometry

89	Geometric probability constant	4.2472965….
90	Circular coverage constant	0.8269933431….
91	Universal coverage constant	Not yet available
92	Hermite's constant	0.7404804897….
93	Tammes' constant	Not yet available
94	Calabi's triangle constant	1.5513875245….
95	Graham's hexagon constant	0.6495190528….
96	Traveling salesman constant	0.521….
97	Moving sofa constant	A=0.09442656084….
98	Beam detection constant	5.1415926536….
99	Heilbronn Triangle constant	H=0.1924500897….
100	Moser's worm constant	Not yet available
101	Rectilinear crossing constant	0.70449881….
102	Maximum irrodius constant	0.2041241452….
103	Magic geometric constant	0.6675276<m<0.6675284

Almost all the constants seem to be irrationals although rigorous proofs are not available. All the constants have numerous fascinating applications, and thus irrational numbers play very important role in studying modern number theory. Details of some interesting constants and some of their applications will be highlighted in 'Ganit Bikash' in near future.

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