A tribute to Emmy Noether on the International Day of Women and Girls in Science

by Abhigyan Ray - Thursday, February 11, 2021

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On the International Day of Women and Girls in Science, this is the inspirational story of Emmy Noether who single handedly shaped the future of mathematics and physics in the early half of the 20th century in the face of insurmountable barricades constructed by antediluvian attitudes towards women in science.

Amalie Emmy Noether was born on March 23, 1882 in the Bavarian university town of Erlangen to Max Noether, a distinguished mathematician and a professor at Erlangen, and Ida Amalia Kaufmann who hailed from a wealthy Cologne family. After basic schooling, Noether wanted to pursue mathematics at university; unfortunately, she was only allowed to audit the courses at Erlangan for a couple of years before she headed to Göttingen for a semester where she was lectured by giants like Minkowski, Hilbert, Klein, and Schwarzschild. On her return she was allowed to matriculate at Erlangan and in 1907 received her D. Phil. summa cum laude for “Über die Bildung des Formensystems der ternären biquadratischen Form” (On the construction of the system of forms of a ternary quartic form) under the tutelage of Paul Gordan, "The King of Invariant Theory", and best known for Clebsch–Gordan pair for the decomposition of combinations of angular momentum vectors.

Noether was the second woman Ph.D. in mathematics in Europe, following Sofia Kovalevskaya who received her degree in Göttingen in 1874, with Karl Weierstraß. She started teaching in the department and came under the influence of the legendary Ernst Fischer who initiated her into Abstract Mathematics. Noether taught for several years without pay before being invited in 1915, to join the University of Göttingen at the invitation of David Hilbert himself. She lectured for other professors and was only allowed to pass her habilitation following the collapse of the Kaiserreich and sweeping university reforms in 1919. Noether became an adjunct professor in 1922, the first female professor in Germany, but despite international recognition, she never received adequate remuneration for her role and nor obtained a permanent position in Göttingen. In Göttingen, Noether developed an international reputation as a pioneering mathematician and the Mathematische Institut became known as the "Mecca of Mathematics." In 1932, she was besmirched by a neighbour as a “Marxist Jewess”, after the astronomical rise of the Nazi party, and eventually she was removed from all teaching duties at the university and was forced to take refuge across the Atlantic and the Rockefeller Foundation helped her obtain a position at the Bryn Mawr College whilst lecturing weekly at the nearby Institute for Advanced Study at Princeton.

Noether’s famous paper, Invariante Variationsprobleme, (English translation by M. A. Tavel) was published in Nachr. d. Konig. Gesellsch. d. Wiss. zu Gottingen, Math-phys. Klasse in 1918. In this paper, Noether proved two different theorems which eponymously came to be known as Noether Theorems. The First Theorem deals with global symmetries, generated by finite Lie groups, and states that these symmetries lead to conserved charges. The Second Theorem applies to local gauge symmetries, infinite dimensional Lie groups, containing arbitrary functions of spacetime like Einstein’s theory of gravity, and shows that these gauge symmetries inevitably lead to relations among the equations of motion. Noether was able to connect these two seemingly unrelated concepts: energy conservation comes from time-

translation invariance and vice versa. Noether explained that the triviality of the energy conservation law follows from the fact that its associated time translational symmetries form a subgroup of such an infinite-dimensional variational symmetry group of the relativistic Hilbert Lagrangian, thus resolving the relativistic energy conundrum. The connection between energy and time was common knowledge, but Noether was the first to theorize a systematic correspondence: symmetries and conservation laws are related, a property widely used in a variety of settings like developing the Yang-Mills theory of the strong interactions among nucleons from isospin symmetry; in the theory of quantum chromodynamics for the strong interactions among quarks and gluons; and in the electroweak theory, where the gauge symmetry must be hidden.

Throughout the 1920s, Noether revolutionised the frontiers in algebra by her studies of ideals, amply helped by a boisterous circle of meritorious graduate students she mentored, the Noether boys, who would go on to become famous mathematicians in their own right. She made seminal contributions to abstract algebra, where she identified a simple, yet elegant, property of number systems, which proved instrumental in the study of arithmetic and geometric phenomena such as prime decomposition and dimension building on the teenage mathematical prodigy, Évariste Galois' revolutionary work. Her fundamental insights led to the development of algebraic topology through the concept of homology groups and combining forces with Emil Artin, Richard Brauer, and Helmut Hasse, she developed the theory of central simple algebras. Noether also gave the first general representation theory of groups and algebras, uniting disparate results from hypercomplex numbers and group representations, and subsuming the structure theory of associative algebras and the representation theory of groups into a single arithmetic theory of modules and ideals in rings satisfying an ascending chain conditions. This single work by Noether heralded the development of modern algebra and the moniker, "Mother of Modern Algebra", was bestowed upon her.

A member of both the Deutsch and American Mathematical Society, Noether became the first woman invited to give a plenary lecture at the 1932 International Congress of Mathematicians (ICM), held in Zürich that year. It wasn't until Karen Uhlenbeck, who would go on to win the Abel Prize in 2019, gave a plenary talk in the 1990 ICM at Kyoto, another woman graced that bully pulpit. Only 18 months after her arrival in the United States, at the age of 53, Noether underwent a routine surgical procedure to remove an ovarian cyst but unfortunately soon serious complications arose and she passed away within days. Immensely generous with her ideas and an excellent and passionate mentor of her students, her death came as a shocker to the scientific world. Eulogies flowed from all across the world with Albert Einstein himself leading the tributes by terming her as "the most significant creative mathematical genius thus far produced since the higher education of women began." From Weyl to Kolmorov to Weiner, giants of mathematics paid tearful tributes to this trailblazing woman who in her short but remarkable life, all the while braving tremendous amounts of gender discrimination and ethnic persecution with her characteristic boldness, left an indelible mark in the history of mathematics and science for aeons to come.
