The unreasonable ubiquity of Mathematics

by Sujatha Ramdorai - Monday, March 03, 2014

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Galileo Galilei

Galileo Galilei (1564-1642) in his book `The Assayer' believed that lasting truth in Physics can only be achieved by Mathematics, and that those who wander without mathematics wander endlessly in a dark labyrinth. He believed that mathematics is the language with which `God has written the Universe'.

This seems uncannily true as the years pass and mathematical ideas and techniques find applications in various other subjects, and the applications of mathematics becomes more pervasive in technology. In this article, we will illustrate this with five examples, the first one in physics, the second in computer science and the remaining ones by way of applications in modern technology and the web.

One of the most striking results in Physics which unifies abstract mathematics with an important physical principle is Noether's theorem. This beautiful theorem, in ordinary language, says that conservation laws in Physics exactly correspond to symmetries in Mathematics. Equivalently:

For every continuous symmetry of the laws of physics, there must exist a conservation law.
Conversely, for every conservation law, there must exist a continuous symmetry.

Emmy Noether

For instance, the conservation law corresponding to symmetries that are invariant under spatial translations (i.e. the laws which are independent of the position) is the Law of Conservation of Momenturm. The conservation law corresponding to time translational symmetry (i.e. the laws that are valid independent of time) is the Law of Conservation of Energy. In mathematics, the concept of continuous symmetry is dealt with in a precise fashion using the language of groups, and Noether's theorem provides a bridge between the conservation laws and the symmetries.

Category theory within mathematics is a highly abstract and formalized branch of mathematics which encapsulates in a succinct manner various phenomena in mathematics. This theory was first introduced by Samuel Eilenberg and Saunders Mac Lane around the middle of the last century, in connection with the study of a branch of mathematics called Algebraic Topology. This high level of abstraction brought it the epithet of 'abstract nonsense'. It led to the evolution of other abstract algebraic theories as well as the branch of mathematics called 'Homological Algebra' which itself has contributed to newer techniques within mathematics. Today category theory is used in theoretical computer science and also in physics.

Let us now turn to technology. Imaging equipments are commonplace today. A key theory that underpins this is the application of a mathematical concept called "Fourier analysis" and the use of Fourier transforms. The Fourier transform can be thought of as a tool that takes points from the "spatial domain" to the image which lies in the ``frequency domain", by decomposing an image into its sine and cosine components. This tool has applications in the areas of image analysis, image filtering, image reconstruction and image compression. Today it is applied in the areas of image recognition, image reconstruction, etc. Discrete Fourier transform is used in the study of digital images. The inverse Fourier transform plays a key role in modern medical imaging techniques.

Saunders MacLane

Control theory is an area that concerns itself with the analysis of control systems that are prevalent in engineering. A major branch of control theory is optimization. Control systems are devised to control an object with the purpose of influencing its behavior so as to achieve a desired goal. Optimization techniques are needed to optimize the behaviour of such systems. The underlying mathematics in control theory is the classical calculus of variations. Control systems are today applied in steering and navigational tools of various transportation systems for example, cars, ships, aircraft. Independently, mathematicians developed a theory called Lie theory to study certain objects Lie groups that occur in nature. To every Lie group is associated an algebra called it as Lie algebra. Today Lie theory is being used in Control theory to study control problems by reinterpreting them as problems on Lie groups. The states of the system under study are ode led by elements of the Lie group and the control aspects as elements of the associated Lie algebra. Control theorists have realized the enormous advantages of such a reinterpretation and it is becoming more common for instance, in the area of robotics and the design of robots.

Samuel Eilenberg

As a final example, we mention two applications of what was developed as abstract or pure mathematics in the internet. All of us today use search engines on the internet in a variety of ways. The underlying mathematical theory for this application comes from a branch of pure mathematics called Ergodic theory, which itself is related to Lie groups and another area called Dynamical statements. Today search engines on the web use a whole under grid of mathematical concepts, encompassing, set theory, logic, linear algebra, graph theory and sophisticated techniques from ergodic theory. In fact, the explosion of data and the accompanying need for analytics is witnessing a whole new development of ``internet mathematics'' which teases out mathematical concepts from abstract mathematics that is relevant for applications on the internet.

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The other application is of course the use of large prime numbers in encryption, especially for exchanging information securely on the internet. The underlying mathematical concept in this case is that of bijective functions, and the use of bijective functions for which one of them is traversed easily while the other requires sophisticated mathematics. An example is the operation of multiplication. The inverse operation is factoring; while large numbers can be easily multiplied today by many computing devices, the converse operation of factoring requires sophisticated results from number theory and there is no known algorithm for efficient factoring of large numbers. However, if a prime factor of a given large number is known, then there is a chance that the converse operation can be simplified and solved. This underlies the need for understanding prime numbers, which in itself is an old area of pure mathematics!

All these examples amply illustrate the diverse applications of mathematical ideas which were studied by mathematicians purely out of interest, and as "mathematics for its own sake"! Galileo was perhaps right in his observation that mathematics holds the key to understanding many mysteries of Nature.

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