Weirstrass' M-Test

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http://gonitsora.com/weirstrass-m-test/

We shall state and prove a very important result in Real Analysis called the Weirstrass' M-Test. The statement of the theorem is give below.

Let $$$$ be a sequence of positive real numbers such that $$mid f_n(x) mid leq M_n$$ for $$x in D$$, $$n in N$$. If the series $$sum M_n$$ is convergent, then $$sum f_n$$ is uniformly convergent on $$D$$.

We prove the result as follows:

If $$m>n$$, we have the relation,

$$mid f_{n+1}(x)+ldots+f_m(x)mid leq M_{n+1}+ldots+M_m,~for~xin D$$.

Since $$sum M_n$$ is convergent so there exists some $$nprime in N$$ such that,

$$mid M_{n+1}+ldots+M_m mid leq epsilon, ~ forall m >nprime.$$

The above relations imply,

$$mid f_{n+1}(x)+ldots+f_m(x)mid < epsilon.$$

It is thus clear now that $$sum f_n$$ is uniformly convergent on $$D$$.

There are other variants of this result, which is left for a later post.

[ad#ad-2]
